Lecture 11
Maher on Physical Probability

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Form of pp statements

Experiments and outcomes

- Let an *experiment* be an action or event such as tossing a coin, weighing an object, or two particles colliding. We distinguish between experiment *tokens* and experiment *types*.
  - An experiment *token* is a physical event at a particular place and time, e.g., a particular toss of a particular coin.
  - An experiment *type* is an abstract categorization to which a token experiment may belong, e.g., "toss of a coin." It doesn't have a place or time.
- Experiments have *outcomes*, such as a coin landing heads. Again there is a type/token distinction.
  - An outcome *token* is a physical event at a particular place and time, e.g., a particular event of a coin landing heads.
  - An outcome *type* is an abstract categorization to which a token outcome may belong, e.g., "landing heads." It doesn't have a place or time.

Statements of pp

- A typical statement of pp:

  The pp of heads on a toss of this coin is 1/2.

  Here the pp relates three things:

  - tossing this coin (an experiment type)
  - the coin landing heads (an outcome type)
  - 1/2 (a number).

- In general, an elementary sentence of pp can be represented in the form:

  The pp of X resulting in O is r

  where X is an experiment type, O is an outcome type, and r is a number.

- *Notation:* "$pp_X(O) = r$" means "the pp of experiment type X having outcome type O is r."

Compatibility with determinism

Pp is compatible with determinism

- Determinism is the view that the state of the world at one time, together with the laws of nature, completely determine the state of the world at all later times.
- The way the concept of pp is used shows that pps can have non-extreme values even if determinism is true.
  - People attribute non-extreme physical probabilities in games of chance, while believing that the outcome of such games is causally determined by the initial conditions.
  - Scientific theories in statistical mechanics, genetics, and the social sciences postulate non-extreme physical probabilities in situations that are believed to be governed by underlying deterministic laws.
  - Some of the most important statistical scientific theories were developed in the 19th century by scientists who believed that *all* events are governed by deterministic laws.

How it is compatible

Determinism implies that, if X is sufficiently specific, then $pp_X(O) = 0$ or 1; but X need not be this specific, in which case $pp_X(O)$ can have a non-extreme value even if the outcome of X is governed by deterministic laws.

Example

A token coin toss belongs to both the following types:

- X: Toss of this coin.
- X': Toss of this coin from such and such a position, with such and such force applied at a such and such a point, etc.

Assuming that the outcome of tossing a coin is governed by deterministic laws, $pp_{X'}(\text{head}) = 0$ or 1; however, this is compatible with $pp_X(\text{head}) = 1/2$.

Differences between pp and ip

1. Ip is logical; pp isn't.
2. The arguments of ip are two propositions; the arguments of pp are an experiment type and an outcome type. Propositions are true or false; experiment and outcome types aren't.
3. Ips exist for practically all pairs of propositions but there are many experiment types X and outcome types O for which $pp_X(O)$ doesn't exist.

   Example: $X =$ placing a die on a table, in whatever way one wants, $O =$ the die is placed with six facing up.
4. Ips often lack numeric values but if a pp exists at all then it has a numeric value.

   Example: $X =$ randomly drawing a ball from an urn of unknown composition, $O =$ the ball drawn is white. We don't know the value of $pp_X(O)$ but it has a numeric value.

Specification

The specification principle (SP)

If it is possible to perform X in a way that ensures it is also a performance of the more specific experiment type X', then $pp_X(O)$ exists only if $pp_{X'}(O)$ exists and is equal to $pp_X(O)$.

Example

X = tossing a normal coin, X' = tossing a normal coin on a Monday, O = the coin lands heads. It is possible to perform X in a way that ensures it is a performance of X' (just toss the coin on a Monday), and $pp_X(O)$ exists, so SP implies that $pp_{X'}(O)$ exists and equals $pp_X(O)$.

Corollary

If it is possible to perform X in a way that ensures it is also a performance of the more specific experiment type X_i, for $i = 1, 2$, and if $pp_{X_1}(O) \neq pp_{X_2}(O)$, then $pp_X(O)$ does not exist.

Example

Let b be an urn that contains only black balls and w an urn that contains only white balls. Let:

$$\begin{aligned} X &= \text{selecting a ball from either } b \text{ or } w \\ X_b &= \text{selecting a ball from } b \\ X_w &= \text{selecting a ball from } w \\ O &= \text{the ball selected is white.} \end{aligned}$$

It is possible to perform X in a way that ensures it is also a performance of the more specific experiment type X_b, likewise for X_w, and $pp_{X_b}(O) = 0$ while $pp_{X_w}(O) = 1$, so the corollary implies that $pp_X(O)$ doesn't exist.

Another look at coin tossing

Let X = tossing a normal coin, O = the coin lands heads.

- If this description of X was a complete specification of the experiment type, then X could be performed with apparatus that would precisely fix the initial position of the coin and the force applied to it, thus determining the outcome. It would then follow from SP that $pp_X(O)$ doesn't exist.
- This consequence of SP is correct.
- So when we say that $pp_X(O)$ does exist, we are tacitly assuming that the toss is made by a normal human without special apparatus that could precisely fix the initial conditions of the toss; a fully explicit specification of X would include this requirement.
- The existence of $pp_X(O)$ thus depends on an empirical fact about humans, namely, the limited precision of their perception and motor control.

Questions

1. Explain the difference between an experiment token and an experiment type; illustrate your answer with an example.
2. What is determinism? If determinism is true, does it follow that all pps are 0 or 1? Explain why, or why not.
3. State SP and its corollary.
4. Let X = tossing a fair coin, O = the coin lands heads. (a) Prove that $pp_X(O)$ doesn't exist, if nothing further is assumed about X. (b) Describe what could be added to X to make it true that $pp_X(O) = 1/2$.
5. If X = placing a die on a table in whatever way one wants and O = the die is placed with six facing up, does $pp_X(O)$ exist? Explain why, or why not.

Reference


Patrick Maher.
Physical probability.
In Clark Glymour, Wei Wang, and Dag Westerståhl, editors, *Proceedings of the 13th International Congress of Logic, Methodology and Philosophy of Science*. Kings College Publications, 2009.
http://patrick.maher1.net/preprints/pp.pdf.