Lecture 7
Keynes on the Principle of Indifference

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The principle of indifference

- *In order that numerical measurement [of probabilities] may be possible, we must be given a number of equally probable alternatives. The discovery of a rule, by which equiprobability could be established, was, therefore, essential. A rule, adequate to this purpose . . . has been widely adopted . . . I prefer to call it The Principle of Indifference.* (41)
- *The Principle of Indifference asserts that if there is no known reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability.* (42)
- Example: If there is no known reason for saying a book is red rather than black then, relative to this knowledge, it is equally probable that the book is red and that the book is black.
- *This rule, as it stands, may lead to paradoxical and even contradictory conclusions.* (42)

Application to arbitrary propositions

Argument

Let a be a proposition about which we have no relevant external evidence; let $\bar{a}$ be the contradictory of a. We've no reason to favor either a or $\bar{a}$ so, by the Principle of Indifference, $p(a) = p(\bar{a})$. Since $p(a) + p(\bar{a}) = 1$, it follows that $p(a) = 1/2$.

Refutation

If . . . having no evidence relevant to the color of this book, we could conclude that 1/2 is the probability of "This book is red," we could conclude equally that the probability of each of the propositions "This book is black" and "This book is blue" is also 1/2. So that we are faced with the impossible case of three exclusive alternatives all as likely as not. (43)

(Probabilities of exclusive alternatives cannot add to more than 1.)

Ratios and constitutions

Application to ratios (49–50)

[Consider] an urn containing black and white balls in an unknown proportion. The Principle of Indifference can be claimed to support the most usual hypothesis, namely, that all possible numerical <u>ratios</u> of black and white are equally probable.

Example

If the urn contains 2 balls, the following would be equally probable:

- 100% of the balls are white.
- 50% of the balls are white.
- 0% of the balls are white.

So each of these has probability 1/3.

Application to constitutions

We might equally well assume that all possible constitutions of the system of balls are equally probable. (50)

(A constitution says which property each individual has.)

Example

If the 2 balls are A and B, the possible constitutions are:

- A and B both white.
- A white, B black.
- A black, B white.
- A and B both black.

If these are equally probable, each has probability $1/4$.

These are inconsistent

Probabilities on these two ways of applying the Principle of Indifference:

	To ratios	To constitutions
100% white	1/3	1/4
50% white	1/3	1/2
0% white	1/3	1/4

Inconsistent!

Using ratios gives induction

Taking ratios as equally probable:

$$\begin{aligned} p(A \text{ white}) &= p(100\% \text{ white}) + p(50\% \text{ white})/2 \\ &= 1/3 + 1/6 = 1/2. \\ p(B \text{ white}) &= 1/2, \text{ similarly.} \end{aligned}$$

$$\begin{aligned} p(B \text{ white} | A \text{ white}) &= \frac{p(A \& B \text{ white})}{p(A \text{ white})} \\ &= \frac{1/3}{1/2} = 2/3. \end{aligned}$$

Learning that A is white raises the probability that B is white.

Uses this rule of probability:

$$p(X|Y) = \frac{p(X \& Y)}{p(Y)}.$$

Using constitutions doesn't give induction

Taking constitutions as equally probable:

$$\begin{aligned} p(A \text{ white}) &= p(A\&B \text{ white}) + p(A \text{ white, } B \text{ black}) \\ &= 1/4 + 1/4 = 1/2. \\ p(B \text{ white}) &= 1/2, \text{ similarly.} \end{aligned}$$

$$\begin{aligned} p(B \text{ white}|A \text{ white}) &= \frac{p(A\&B \text{ white})}{p(A \text{ white})} \\ &= \frac{1/4}{1/2} = 1/2. \end{aligned}$$

Learning A is white doesn't change the probability that B is white.

Analogous results hold when there are more than two balls.

Questions

1. What is the Principle of Indifference?
2. The Principle of Indifference has sometimes been used to argue that if a is a proposition about which we have no relevant external evidence then the probability of a is $1/2$. Is this conclusion correct? Justify your answer.
3. Suppose we are given that an urn contains two balls, each of which may be either black or white. Describe two different ways that the Principle of Indifference can be applied to this situation. What is the probability of the following propositions according to these two ways?
   (a) Both balls are white.
   (b) One ball is white and one is black.
   (c) Both balls are black.
   (d) The second ball is white.
   (e) The second ball is white given that the first is white.

Reference

John Maynard Keynes.
A Treatise on Probability.
Macmillan, 1921.
On Google books.
Numbers in parentheses are page numbers of this book.