Lecture 23
Einstein on the Relativity of Length and Time

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Scientific Thought II
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Review

- Clocks at A and B are synchronous if, when a ray of light is sent from A to B and reflected back to A,

$$t_B - t_A = t'_A - t_B$$

$t_A =$ reading on the clock at A when the ray leaves A
$t_B =$ reading on the clock at B when the ray arrives at B
$t'_A =$ reading on the clock at A when the ray returns to A.

- *The "time" of an event is the reading obtained simultaneously from a clock at rest that is located at the place of the event, this clock being synchronous . . . with a specified clock at rest.* (127)
- *We have defined time by means of clocks at rest in the rest system; because the time just defined is related to the system at rest, we will call it "the time of the rest system."*

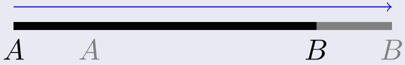
Relativity of simultaneity

Experimental setup (129)

- A rod, with ends marked A and B, is moving (in the rest system) with constant velocity in the direction from A to B.
- There is a clock at each end of the rod. These clocks always tell rest system time, i.e., they agree with clocks at the same place that are at rest in the rest system and synchronized with a specified clock at rest in the rest system.
- Since both clocks on the rod tell rest system time, they are synchronous in the rest system.

Moving clocks not synchronous in moving system (129–30 simplified)

- Suppose observers moving with the rod apply the procedure with light rays to determine whether the clocks on the rod are synchronous. We will describe what happens using rest system coordinates.
- Light traveling from A to B travels further than the length of the rod because the rod is moving in the same direction.

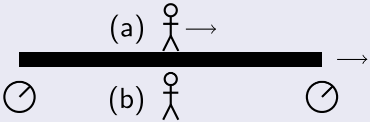


- Light traveling from B to A travels less than the length of the rod because the rod is moving in the opposite direction.
- The velocity of light in the rest system is a constant c, regardless of the velocity of the source (why?), so light takes longer to get from A to B than from B to A.
- Since the clocks tell rest-system time, $t_B - t_A > t'_A - t_B$. The clocks aren't synchronous in the moving system.

Simultaneity is relative

- Suppose lightning strikes both ends of the rod. Suppose the times showing on the clocks when the lightning struck were the same at each end.
- Since the clocks are synchronous in the rest system, the lightning strikes were simultaneous in the rest system, i.e., they happened at the same time.
- Since the clocks are *not* synchronous in the moving system, the fact that these clocks showed the same time when the lightning strikes occurred implies that the lightning strikes were *not* simultaneous.
- *Thus we see that we cannot ascribe absolute meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system.* (130)

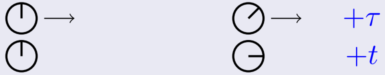
- Consider a rod moving with velocity v in the rest system. We've seen two methods of measuring its length.
  (a) *In the moving system*, lay out a measuring rod.
  (b) *In the rest system*, determine where the ends of the rod are located at some one time, using synchronized clocks at rest in the rest system, then use a measuring rod to find the distance between these points.



- Suppose the length of the rod in the moving system is l. Einstein deduced from his two principles that its length in the rest system is $l\sqrt{1-(v/c)^2}$.
- Thus the rod is *shorter* in the rest system than in the moving system.

Relativity of time (138–39)

- Suppose a clock, constructed the same as those that correctly tell rest system time, is moving with velocity v in the rest system.
- We read what this moving clock says and also record the rest system time where the clock is. Later we again read what the moving clock says and record the rest system time where the clock now is.



- Let τ be the time that has passed according to the moving clock and t the time that has passed according to the rest system clocks. Einstein deduced from his two principles that $\tau = t\sqrt{1-(v/c)^2}$.
- Thus the moving clock is *running slow* with respect to rest system time.

Switching perspective

- We've seen that moving bodies are contracted in the rest system and moving clocks are running slow in the rest system.
- *Question:* In the moving system, are bodies at rest in the rest system expanded, contracted, or neither? Do clocks at rest in the rest system run fast, slow, or neither?
- *Answer:* We could call the moving system "the rest system," in which case things formerly said to be at rest in the rest system would be moving. This change can't change what laws hold (why?). Hence bodies at rest in the rest system are contracted in the moving system and clocks at rest in the rest system are running slow in the moving system.
- *It is clear that the same results apply for bodies at rest in the "rest" system when considered from a uniformly moving system.* (138)

Questions

1. Let a rigid rod with endpoints A and B be moving in the rest system with constant velocity in the direction from A to B. Suppose there are clocks on the rod at A and B that tell rest-system time. Prove that these clocks aren't synchronous in the moving system.
2. Einstein argued that clocks synchronized in one coordinate system aren't synchronized in a coordinate system moving relative to the first. How does it follow from this that "we cannot ascribe *absolute* meaning to the concept of simultaneity"?
3. A rod is traveling in the rest system, in the direction of its length, at half the speed of light. If the rod is 2 meters long in the moving system, how long is it in the rest system? Show how you obtained your answer.
4. In the moving system, are clocks at rest in the rest system running fast, slow, or neither? Justify your answer.

Quantitative treatment of simultaneity (129–30, not on exam)

r_{AB} = the length of the moving rod in the rest system,
v = velocity of the rod. Using rest system coordinates:

- For light traveling from A to B,

$$c = \frac{\text{distance traveled}}{\text{time taken}} = \frac{r_{AB} + v(t_B - t_A)}{t_B - t_A}.$$

  Rearranging gives $t_B - t_A = r_{AB}/(c - v)$.

- For light traveling from B to A,

$$c = \frac{\text{distance traveled}}{\text{time taken}} = \frac{r_{AB} - v(t'_A - t_B)}{t'_A - t_B}.$$

  Rearranging gives $t'_A - t_B = r_{AB}/(c + v)$.

- Therefore, $t_B - t_A > t'_A - t_B$.

Reference

Albert Einstein.
On the electrodynamics of moving bodies.
In John Stachel, editor, *Einstein's Miraculous Year: Five Papers That Changed the Face of Physics*. Princeton University Press, 1998.
Numbers in parentheses are page numbers of this edition.
I corrected the translation and modernized notation in places.