Explication of Inductive Probability

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Introduction

Inductive probability

- "Probability" in ordinary English has two senses, which I call *inductive probability* (ip) and *physical probability* (pp).
- The arguments of ip are two propositions; we speak of the probability *of* proposition A *given* proposition B; I'll write this as $ip(A|B)$.
- *Example:* Let $E =$ a coin is either two-headed or two-tailed and is about to be tossed, $H =$ the coin will land heads; then $ip(H|E) = 1/2$.
- A function is *logical* if all true elementary sentences for it are analytic. *Example:* A function defined by specifying its values for all possible arguments.
- $ip(A|B)$ is fixed once A and B are fixed. Hence ip is logical.

Explication

- Many ips don't have a numeric value. *Example:* The ip that humans evolved in Africa, given what I know.
- This shouldn't be surprising; ip is a concept of ordinary language and many concepts of ordinary language are vague.
- It is difficult to reason rigorously about vague concepts, especially in complex situations. Explication is a methodology for mitigating this difficulty.
    - *Explicandum:* the pre-existing vague concept.
    - *Explicatum:* a precise concept that is similar enough to the explicandum that it can be used in place of it for some envisaged purposes.
    - *Explication:* finding an explicatum for an explicandum.
- I'll discuss how to explicate the concept of ip.

General considerations

Domain of the explicatum

- $ip(A|B)$ is meaningful for enormously diverse A and B. It isn't feasible to construct an explicatum with such a diverse domain. This is also not necessary.
- So the first step is to specify the pairs of propositions A and B for which we'll explicate $ip(A|B)$.
- I'll assume we aim to explicate all ips of the form $ip(A|B.K)$, where A and B are in some algebra $\mathcal{A}$ of propositions and K is a fixed proposition. K is called "background evidence."
- *Example:* $K =$ a coin will be tossed twice and land either heads or tails on each toss, $\mathcal{A} =$ the algebra generated by H_1 and H_2, where $H_i =$ the coin lands heads on the ith toss.

Form of the explicatum

- The explicatum will be a function p that takes two elements of $\mathcal{A}$ as arguments and has real numbers $p(A|B)$ as its values.
- Aim is for $p(A|B)$ to be a good explicatum for $ip(A|B.K)$.
- p will be defined by axioms that specify $p(A|B)$ for all $A, B \in \mathcal{A}$. The specification won't depend on contingent facts, so p is logical.
- Among the axioms are ones that ensure p obeys the mathematical laws of probability. Motivation:
    - When ips have numeric values they satisfy these laws.
    - A good explicatum is fruitful and simple.

Axioms that ensure p satisfies the laws of probability

For all $A, B, C, D \in \mathcal{A}$:

1. $p(A|B) \geq 0$.
2. $p(A|A) = 1$.
3. $p(A|B) + p(\sim A|B) = 1$, provided $B.K$ is consistent.
4. $p(A.B|C) = p(A|C)\, p(B|A.C)$.
5. If $A.K \Leftrightarrow C.K$ and $B.K \Leftrightarrow D.K$ then $p(A|B) = p(C|D)$.

Noteworthy features:

- $p(A|B)$ defined even for inconsistent B.
- Takes account of background evidence.

Carnap's Basic System

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- The preceding axioms only fix the value of $p(A|B)$ if $B.K \Rightarrow A$ or $B.K \Rightarrow \sim A$.
- Hence additional axioms must be added to complete the definition of p. These must depend on the content of K and the propositions in $\mathcal{A}$.
- I'll present an example of such axioms from Carnap's "Basic System of Inductive Logic" (1971, 1980).

Domain of the explicatum

- There is a denumerable set of individuals a_1, a_2, ...
- A *modality* is a type of property, e.g., color, shape, substance.
- A *family of properties* is a set of properties that belong to one modality, are mutually exclusive, and jointly exhaustive.
- A *primitive property* is a property that isn't defined in terms of other properties in our analysis. Carnap took the primitive properties to be the elements of a finite family of properties. My notation: $F_1, \ldots, F_k$.
- Grue combines two modalities and hence can't be one of Carnap's primitive properties.
- An *atomic proposition* ascribes a primitive property to an individual. Notation: $F_i a_j$.
- $\mathcal{A}$ = the algebra generated by the atomic propositions, K = an analytic proposition.

Additional axioms

$p(A) = p(A|T)$, where T is analytic.
$E =$ any sample proposition, i.e., finite conjunction of atomic propositions for different individuals.

6. $p(E) > 0$.
7. $p(E)$ isn't changed by permuting individuals.
8. $p(F_i a_n | E.F_i a_m) > p(F_i a_n | E)$,
   provided E doesn't involve a_m or a_n.
9. If a is any individual not involved in E then $p(F_i a | E)$ depends only on the number of individuals mentioned in E and the number that E says have F_i.

The preceding axioms imply:

$\lambda\gamma$ theorem

If $k > 2$ then there exist $\lambda > 0$ and $\gamma_1, \ldots, \gamma_k \in (0, 1)$ such that

$$p(F_i a|E) = \frac{s_i + \lambda\gamma_i}{s + \lambda}$$

where E is a sample proposition for a sample of s individuals, s_i is the number of individuals to which E ascribes F_i, and a is any individual not involved in E.

To get numeric values for p we must fix the values of λ and the γ_i.

How to fix the γ_i

- The $\lambda\gamma$ theorem implies $p(F_i a) = \gamma_i$. Thus γ_i needs to be a good explicatum for $ip(F_i a|T)$.
- The *attribute space* for the F_i is the logical space whose points are the most specific properties of the relevant modality.
- Each F_i corresponds to a region of the attribute space. Carnap proposed that γ_i be set equal to the proportion of the attribute space that corresponds to F_i.
- *Example:* If the modality is color and F_1 is red, γ_1 is the proportion of a color solid whose points are a kind of red.

How to fix λ

Carnap's approach:

- He considered different values of λ in a variety of examples and concluded that, for p to agree with ip (my terminology), λ should be not much less than 1 or much greater than 2.
- Since integer values are simplest, he further concluded that λ should be set equal to either 1 or 2.

I add:

- De Finetti's representation theorem implies that we can think of the a_i as tokens of an experiment type with an unknown pp of giving an outcome of type F_i.
- If $\gamma_i = 1/2$ then the expected value of this pp must be $1/2$. It is then natural to explicate the a priori ip distribution for this pp as uniform from 0 to 1.
- This and the $\lambda\gamma$ theorem implies $\lambda = 2$.

Spurious criticisms of Carnap

Quotations are from Hájek (2007).

Arbitrariness

Is there a correct setting of λ, or said another way, how "inductive" should the confirmation function be? The concern here is that any particular setting of λ is arbitrary in a way that compromises Carnap's claim to be offering a logical notion of probability.

- The choice of λ isn't arbitrary; it's designed to ensure that *p* is a good explicatum for ip and I've argued $\lambda = 2$ is best for this.
- Even if it was arbitrary, *p* would still be logical in the sense Carnap claimed because its values are specified by its definition in a way that doesn't depend on contingent facts.

Axioms of symmetry

Significantly, Carnap's various axioms of symmetry are hardly logical truths. Moreover, Fine (1973) argues that we cannot impose further symmetry constraints that are seemingly just as plausible as Carnap's, on pain of inconsistency.

- I stated just one axiom of symmetry. Like the other axioms, it is part of the definition of p and hence a logical truth.
- There are uncountably many probability functions that satisfy all the constraints that Fine claimed are not jointly satisfiable.
- One of Fine's constraints is not something an explicatum for ip should satisfy. It implies that all γ_i have the same value. It also implies that, when there are multiple families of properties, the explicatum is insensitive to analogies between individuals that the evidence says differ in any respect.

Syntactic approach

Another Goodmanian lesson is that inductive logic must be sensitive to the meanings of predicates, strongly suggesting that a purely syntactic approach such as Carnap's is doomed.

- Carnap's Basic System assigns p values to pairs of propositions, not expressions; hence it isn't a syntactic approach.
- Hájek's objection seems to be that, because of its allegedly syntactic approach, Carnap's IL is unable to distinguish properties like grue from normal properties like green and blue. But Carnap did distinguish between them, as we saw.

No canonical language

Finding a canonical language seems to many to be a pipe dream, at least if we want to analyze the "logical probability" of any argument of real interest—either in science, or in everyday life.

- Hájek doesn't explain what a canonical language is or why he thinks Carnap is committed to it.
- One of Carnap's central philosophical principles was that there is no uniquely correct language (*Principle of Tolerance*).
- Perhaps Hájek meant that Carnap's IL can only deal with propositions expressed in a formal language and no such language can express "any argument of real interest." But neither part of this is true.

Total evidence not well defined

If one's credences are to be based on logical probabilities, they must be relativized to an evidence statement, e. But which is it to be? Carnap's recommendation is that e should be one's total evidence ... However, when we go beyond toy examples, it not clear that this is well-defined.

- I'll take the objection to be that a person's total evidence is too vague or complex to be represented in Carnap's IL.
- We can *explicate* a person's total evidence for a given situation with a relatively precise proposition.
- Alternatively, we can denote a person's total evidence as "K," without attempting to articulate all that it contains, and explicate ips conditional on K.

Foundationalism

The total evidence criterion goes hand in hand with positivism and a foundationalist epistemology according to which there are such determinate, ultimate deliverances of experience. But perhaps learning does not come in the form of such "bedrock" propositions, as Jeffrey (1992) has argued.

- Carnap (1936, 1963) denied there are "bedrock" propositions.
- But $ip(E|E) = 1$ so, if I use ips given my evidence to guide actions, I will act as if my evidence is certainly true.
- Carnap never explained how to reconcile these things.
- *My solution:* What counts as our evidence is only sufficiently certain that it can be treated as certain in the present context.

Legitimate criticism of Carnap

Unrealistically simple

The explicata developed by Carnap have very simple domains and, as a result, aren't applicable to most situations of real interest.

- This doesn't show that the methodology of explicating ip is similarly limited. Bayesian statisticians have developed probability models for a wide variety of realistic domains; these are best interpreted as explications of ip.
- An explication of ip for an artificially simple domain can help clarify fundamental questions about confirmation and resolve philosophical paradoxes (Maher 2004).

Conclusion

- Explication of ip is a valuable methodology for reasoning about ip.
- The explication of Carnap's that I presented is a creditable simple example of such an explication.