Lecture 7
Carnap's Inductive Logic

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Introduction

- Rudolf Carnap (1891–1970) worked on explicating inductive probability from the early 1940s until his death in 1970.
- His last and best explicatum was published posthumously in two parts, in 1971 and 1980, with the title "A Basic System of Inductive Logic."
- Most discussions of Carnap's inductive logic only talk about his earlier systems that are not as good.
- I'll present one special case from the "Basic System."
- I won't always state it exactly the way Carnap did. E.g.:
  - Carnap here applied probabilities to sets of possibilities (called propositions); I'll use sentences.
  - Carnap called his explicatum $\mathcal{C}$; I'll call it p.

The language $\mathcal{L}$

Individual constants (1971 p. 43)

- There is a set of basic objects that can be talked about in $\mathcal{L}$; these are called *individuals*. They are denoted by *individual constants* a_1, a_2, ...
- Different individual constants are assumed to denote different individuals; "$a_2 = a_5$" is contradictory.

Modalities (1971 pp. 43–45)

A modality is (here) a kind of property. Examples:

- Color (red, blue, etc.)
- Shape (spherical, cubical, etc.)
- Substance (iron, stone, wood, etc.)
- Age in years (0, 1, 2, ...)

Primitive predicates (1971 pp. 43, 121)

- A *family of properties* is a set of properties that belong to one modality, are mutually exclusive, and jointly exhaustive. Examples:
  - {red, orange, yellow, green, blue, indigo, violet, another color}
  - {black, non-black}
  - {less than 1 year old, 1 year old, 2 years old, . . . }
- The *primitive predicates of $\mathcal{L}$* denote the elements of one family of properties. These predicates are F_1, F_2, . . . , F_k.

Grue (1971 pp. 73–74)

- Nelson Goodman defined the predicate "grue" as follows: It applies to things examined before time t iff they are green, and to things not examined before t iff they are blue.
- No primitive predicate of $\mathcal{L}$ can mean "grue" because that concept is a combination of two modalities.

Sample descriptions

- An *atomic sentence* consists of a primitive predicate followed by an individual constant.
  - "$F_2 a_5$" means that a_5 has property F_2.
- A *sample* is a finite set of individuals.
- A *sample description* is a conjunction of atomic sentences, one for each individual in the sample.
  - "$F_2 a_1 . F_5 a_2$" is a sample description for the sample $\{a_1, a_2\}$.
- We allow the empty set to count as a sample. We deem an analytic sentence to be a sample description for the empty set.

The function p

As in Lecture 3, p is here a function that takes pairs of sentences of $\mathcal{L}$ as arguments and satisfies A1–A5. We now adopt the following additional axioms. Here, $p(A)$ means $p(A|T)$, where T is analytic.

A6–A9

Let S be a sample description.

A6. *Regularity:* $p(S) > 0$.

A7. *Symmetry:* $p(S)$ isn't changed by permuting individual constants.

E.g., $p(F_2a_1.F_5a_2) = p(F_2a_2.F_5a_1)$.

A8. *Instantial relevance:* $p(F_ia_m|F_ia_n) > p(F_ia_m)$, for all i, m, n.

A9. *λ-condition:* If S doesn't involve a then $p(F_ia|S)$ depends only on the number of individuals mentioned in S and the number that S says have F_i.

E.g., $p(F_1a_3|F_1a_1.F_2a_2) = p(F_1a_3|F_1a_1.F_3a_2)$.

Justification of A6–A9

The justification must be that ip satisfies these axioms.

A6. Ip satisfies it if none of the F_i are infinitely precise, e.g., specifying the exact wavelength of light reflected by an object.

A7. Ip satisfies it if the individual constants carry no information about which F_i the individual has.

A8. This is a characteristic property of ip: learning from experience.

A9. This is commonly assumed; we typically take the relative frequency of a property in a sample as what is relevant to predicting whether other individuals have that property.

It can fail if, for example, F_1 is more similar to F_2 than to F_3.

The $\lambda\gamma$ theorem (Carnap 1980, §19)

If p satisfies A1–A9 and $k > 2$ then there exist $\lambda > 0$ and $\gamma_1, \ldots, \gamma_k \in (0, 1)$ such that:

$$p(F_i a|S) = \frac{n_i + \lambda\gamma_i}{n + \lambda}, \text{ for all } i = 1, \ldots, k.$$

- a is any individual constant not in the sample proposition S.
- n is the number of individual constants in S.
- n_i is the number of individuals that S says have F_i.

Example

Suppose $\gamma_1 = 1/4$ and $\lambda = 2$; then

$$p(F_1 a_4|F_1 a_1.F_2 a_2.F_3 a_3) = \frac{1 + 2/4}{3 + 2} = \frac{3}{10}.$$

Extension to $k = 2$ requires a further assumption. (1980 p. 98)

How to fix the γ_i

- By the $\lambda\gamma$ theorem, $\gamma_i = p(F_i a)$, i.e., the a priori probability that something has F_i.
- Laplace assumed this is $1/2$. Keynes showed that's not always right.
- Carnap suggested the following approach:
  - The *attribute space* for the F_i is the logical space whose points are the most specific properties of the relevant modality. Example: Color space. The F_i denote regions of this space. (1971 pp. 43–45)
  - γ_i is the proportion of the total attribute space that is taken up by F_i. (1980 pp. 8–12, 33–34)
- Example: If F_1 means "red," and red occupies $1/20$ of the volume of a color space, we could set $\gamma_1 = 1/20$ (assuming the object is monochromatic).

The meaning of λ

$$p(F_i a|S) = \frac{n_i + \lambda\gamma_i}{n + \lambda} = \left(\frac{n}{n+\lambda}\right)\frac{n_i}{n} + \left(\frac{\lambda}{n+\lambda}\right)\gamma_i.$$

- So $p(F_i a|S)$ is a mixture of an empirical factor, n_i/n, and a logical factor, γ_i.
- The larger λ is, the more weight is put on the logical factor, and the slower one learns from experience.

How to fix λ (my approach; Carnap 1980 pp. 111–119 is different)

Laplace's derivation of the Rule of Succession is reasonable for F_i if $\gamma_i = 1/2$ and there is a pp of F_i. So we should have:

$$\frac{n + \lambda/2}{n + \lambda} = \frac{n+1}{n+2}.$$

This implies $\lambda = 2$.

We have now completed an explication of ip!

Questions

1. For each of the following, say whether it is implied by A1–A9, incompatible with A1–A9, or neither. Justify your answers.
   (a) $p(F_1a_1.F_2a_1) > 0$.
   (b) $p(F_1a_1.F_2a_2) > 0$.
   (c) $p(F_1a_1.F_1a_2) > 0$.
   (d) $p(F_1a_1.F_2a_2) = p(F_2a_1.F_1a_2)$.
   (e) $p(F_1a_1.F_1a_2) < p(F_1a_1.F_1a_3)$.
   (f) $p(F_2a_1|F_2a_2) = p(F_2a_1)$.
   (g) $p(F_1a_1|F_2a_2) = p(F_1a_1|F_3a_3)$.
   (h) $p(F_1a_1|F_2a_2.F_3a_3) > p(F_1a_2|F_4a_4.F_5a_5)$.
2. State the $\lambda - \gamma$ theorem.
3. What significant quantity is γ_i equal to? What was Carnap's suggestion for fixing its value?
4. What effect does increasing or decreasing the value of λ have?
5. What value of λ does Maher favor? What is his justification?

Hájek's criticisms of Carnap's inductive logic

- Hájek (2003, sec. 3.2) raises many objections to Carnap's inductive logic. They aren't original with him, but are rather a digest of things many philosophers have said.
- I think none of these is a good objection, but since they are often repeated, it is worthwhile to go through them and say what's wrong with them.

Arbitrariness

The whole point of the theory of logical probability is to explicate ampliative inference, although given the apparent arbitrariness in the choice of language and in the setting of λ—thus, in the choice of confirmation function—one may wonder how well it achieves this.

- $\mathcal{L}$ and λ are chosen to make p a good explicatum for ip. To the extent that the choice is determined by the criteria for a good explicatum, it isn't arbitrary.
- There can be more than one good explicatum for a concept. Choice between these is arbitrary.
- To show that an explicatum is unsuccessful, one must show that it doesn't satisfy one or more of the criteria for a good explicatum. Since non-arbitrariness isn't one of these criteria, it is irrelevant whether p is arbitrary or not.

Universal generalizations

A universal statement in an infinite universe always receives zero confirmation, no matter what the (finite) evidence. Many find this counterintuitive, since laws of nature with infinitely many instances can apparently be confirmed. Earman (1992) discusses the prospects for avoiding the unwelcome result.

- As Hájek acknowledges, there are known ways of modifying Carnap's systems to avoid this result. So this is only an objection to the particular explicata Carnap developed, not to the general project he was pursuing.
- Carnap's explicata are useful when we are concerned with finite populations.

Axioms of symmetry

Significantly, Carnap's various axioms of symmetry are hardly logical truths. More seriously, we cannot impose further symmetry constraints that are seemingly just as plausible as Carnap's, on pain of inconsistency—see Fine (1973, 202).

- These axioms are part of the definition of p, hence analytic, and in that sense logical truths.
- Fine (pp. 189–193) listed five conditions he claimed were inconsistent. One of them (L6) implies that all γ_i have the same value. This is not desirable, in general.
- The same condition implies that there is no reasoning by analogy when individuals are known to differ. This is not desirable, ever.
- I don't see why Fine's conditions are inconsistent and I haven't been able to follow his proof (pp. 205–206).

Syntactic approach

Another Goodmanian lesson is that inductive logic must be sensitive to the meanings of predicates, strongly suggesting that a purely syntactic approach such as Carnap's is doomed.

- In the Basic System, predicates are organized in families that belong to the same modality, and the γ_i depend on the width of the predicates.
- These are semantic requirements (they involve the meanings of the predicates), not syntactic ones.
- Thus Carnap did *not* have a "purely syntactic approach" and his inductive logic *is* sensitive to the meanings of predicates.

No canonical language

Finding a canonical language seems to many to be a pipe dream, at least if we want to analyze the "logical probability" of any argument of real interest—either in science, or in everyday life.

- Carnap didn't believe in, or assume, a "canonical language." On the contrary, one of his central philosophical convictions was that different languages are permissible.
- Perhaps Hájek meant to say that no formalized language could fully express all the evidence available in a real situation. That is dealt with in the next objection.

Total evidence isn't well defined

If one's credences are to be based on logical probabilities, they must be relativized to an evidence statement, e. But which is it to be? Carnap's recommendation is that e should be one's total evidence . . . *However, when we go beyond toy examples, it not clear that this is well-defined. Suppose I have just watched a coin toss, and thus learned that the coin landed heads. Perhaps "the coin landed heads" is my total evidence? But I also learned a host of other things . . . Perhaps, then, my total evidence is the infinite intersection of all these propositions, although this is still not obvious—and it is not something that can be represented by a sentence in one of Carnap's languages, which is finite in length.*

The *ordinary language* concept of total evidence is indeed vague. We *explicate it* for a given situation using the sentences of a formal language. The explication is good if it captures the factors that are relevant for the problem we are concerned with. What we cite in real arguments is always expressible as a sentence.

Foundationalism

The total evidence criterion goes hand in hand with positivism and a foundationalist epistemology according to which there are such determinate, ultimate deliverances of experience. But perhaps learning does not come in the form of such "bedrock" propositions, as Jeffrey (1992) has argued—maybe it rather involves a shift in one's subjective probabilities across a partition, without any cell of the partition becoming certain.

- For E to be a good explicatum for a person's evidence, what is needed is only: (i) E is probable enough that it can be treated as certain for the purposes at hand; (ii) E isn't inferred from other propositions that are under consideration. This doesn't assume any "ultimate deliverances of experience." (Maher 1996 pp. 158–162)
- Non-propositional evidence can be represented in Carnapian inductive logic with a variable that is treated like a proposition. (Pearl 1990)

Science may overthrow inductive logic

By Carnap's lights, the degree of confirmation of a hypothesis depends on the language in which the hypothesis is stated ... But scientific progress often brings with it a change in scientific language (for example, the addition of new predicates ...) ... Thus, the growth of science may overthrow any particular confirmation theory. There is something of the snake eating its own tail here, since logical probability was supposed to explicate the confirmation of scientific theories.

- The objection seems to be that inductive logic assumes the scientific theories whose probability it is supposed to explicate. That is false, since all theorems of inductive logic are analytic.
- "The growth of science" can't change $ip(H|E)$, for any fixed H and E. Therefore, the growth of science can't change whether p is a good explicatum for ip. In that sense, it is false that "the growth of science may overthrow any particular confirmation theory."

Question

6. Are the following criticisms of Carnap's inductive logic sound? Justify your answers.
   (a) The whole point of the theory of logical probability is to explicate ampliative inference, although given the apparent arbitrariness in the choice of language and in the setting of λ—thus, in the choice of confirmation function—one may wonder how well it achieves this.
   (b) If one's credences are to be based on logical probabilities, they must be relativized to an evidence statement, *e*. But which is it to be? Carnap's recommendation is that *e* should be one's total evidence . . . However, when we go beyond toy examples, it not clear that this is well-defined.
   (c) The total evidence criterion goes hand in hand with positivism and a foundationalist epistemology according to which there are such determinate, ultimate deliverances of experience. But perhaps learning does not come in the form of such "bedrock" propositions, as Jeffrey (1992) has argued.
   (d) Scientific progress often brings with it a change in scientific language . . . Thus, the growth of science may overthrow any particular confirmation theory. There is something of the snake eating its own tail here, since logical probability was supposed to explicate the confirmation of scientific theories.

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