Lecture 1
The Concept of Inductive Probability

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Two concepts of probability

Example 1

You know that a coin is either two-headed or two-tailed but you have no information about which it is. The coin is about to be tossed. What is the probability that it will land heads? There are two natural answers:

- 1/2
- Either 0 or 1 but I don't know which.

Terminology

- Inductive probability: The meaning of "probability" in which "1/2" is the natural answer.
- Physical probability: The meaning of "probability" in which "0 or 1" is the natural answer.

Contrasts

- *Inductive probability:*
  - Relative to available evidence
  - Doesn't depend on unknown facts about the world
- *Physical probability:*
  - Not relative to available evidence
  - A fact about the world, often unknown

Example 2

Suppose you see that the coin in the previous example has a head on one side.

- The inductive probability of the coin landing heads on the next toss is now 1 (it has changed).
- You now know the physical probability of the coin landing heads is also 1 (but it hasn't changed).

Example 3

At the end of *Antony and Cleopatra*, Cleopatra is found dead with evidence suggesting snake bite. Caesar says:

> *Most probable that so she died; for her physician tells me she hath pursued conclusions infinite of easy ways to die.*

- This probability is inductive, not physical (it is relative to the evidence).
- Although the probability is high, it doesn't have a precise numeric value.

Subjective probability

The *subjective probability* of H for person X at time t is the degree to which X believes H at time t.

Inductive probability isn't subjective probability

1. Dictionaries don't give degree of belief as a meaning of "probability."
2. If ordinary people are asked what "probability" means, they won't say it means a person's degree of belief.
3. If inductive probability is subjective probability, people who assert different values for an inductive probability aren't contradicting one another. But they are.
4. If inductive probability is subjective probability, claims about inductive probability can be supported by evidence about the speaker's psychological state. But they can't.

Expression vs assertion

De Finetti was one of the most influential advocates of subjective probability. Some of his statements suggest this argument:

> *Our assertions about inductive probabilities express our degrees of belief, so these assertions can only mean that we have these degrees of belief.*

That argument is invalid.

- All sincere intentional assertions *express* our beliefs but most such assertions are not *about* our beliefs. We need to distinguish between the content of an assertion and the state of mind which that assertion expresses.
- Analogy: If I say it's raining, I'm expressing my belief that it's raining but I'm not asserting that I have this belief; I'm asserting that it's raining.

Questions

1. Describe a situation in which the inductive probability of a die landing six is (a) the same as its physical probability; (b) different to its physical probability.
2. State two differences between inductive and physical probability.
3. Is subjective probability the same as inductive probability? Justify your answer.
4. "Since our assertions about inductive probabilities express our degrees of belief, they can have no meaning other than that we have these degrees of belief." Is this a sound argument? Justify your answer.

Rational degree of belief

Hypothesis 1

The inductive probability of H given E is the degree of belief in H that is rational for a person whose evidence is E.

Counterexample

Suppose that X is a competitor in a sports event and knows that he will perform better if he has a high degree of belief that he will win. Then it may be rational for X to have a high degree of belief that he will win, even if the inductive probability of this, given X's evidence, is low.

Hypothesis 2

- The inductive probability of H given E is the degree of belief in H that is *epistemically* rational for a person whose evidence is E.
- A belief is epistemically rational if it is conducive towards epistemic goals such as truth and avoidance of error *(instrumental conception of epistemic rationality)*.

Counterexample

Suppose:

- X knows that a superior being offers tremendous knowledge to those who have a high degree of belief in H.
- The inductive probability of H given X's evidence is low.

Then the degree of belief in H that is (instrumentally) epistemically rational for X may be high, though the inductive probability of H given X's evidence is low.

Hypothesis 3

- The inductive probability of H given E is the degree of belief in H that is epistemically rational for a person whose evidence is E.
- A belief is epistemically rational if it is supported by the person's evidence *(evidential conception of epistemic rationality)*.

- X's degree of belief in H is supported by X's evidence if it agrees with the inductive probability of H given X's evidence.
- So Hypothesis 3 is *trivially true*.

Conclusion

The identification of inductive probability with rational degree of belief is false on some natural interpretations and in the sense in which it is true it is trivial.

Questions

5. Is the inductive probability of H given E the same as the degree of belief in H that would be rational for a person whose evidence is E? Justify your answer.
6. What is the instrumental conception of epistemic rationality? Is the inductive probability of H given E the same as the degree of belief in H that would be instrumentally epistemically rational for a person whose evidence is E? Justify your answer to the latter question.
7. What is the evidential conception of epistemic rationality? Is the inductive probability of H given E the same as the degree of belief in H that would be evidentially epistemically rational for a person whose evidence is E? Justify your answer to the latter question.

Logical probability

The term "logical probability" is often used. We'll consider what it means and whether it is the same as inductive probability.

Logical probability as rational degree of belief

- "Logical probability" is often taken to mean "uniquely rational degree of belief" (Salmon, Skyrms, Gillies).
- But "rational degree of belief" is ambiguous, as we've seen.
- On some interpretations, inductive probability isn't the same as rational degree of belief.
- The only sense in which it is the same is the trivial one in which "rational degree of belief" means a degree of belief equal to the inductive probability.
- So it is wrong or trivial to identify inductive probability with logical probability, on this conception of "logical probability."

Logically determinate sentences

- Some sentences are true merely due to the meanings of the terms occurring in them. These are said to be analytic.
  - E.g.: "No bachelor is married."
- Some sentences are false due merely to the meanings of the terms occurring in them. These are said to be contradictory.
  - E.g.: "There is a married bachelor."
- For other sentences, whether they are true or false isn't determined solely by meanings; it depends also on facts. These are said to be synthetic.
  - E.g.: "Objects fall to the earth with an acceleration of 32 ft/sec^2."
- A logically determinate sentence is a sentence that is either analytic or contradictory.
- Equivalent definitions of logically determinate sentence:
  - Its truth value (true or false) is determined by meanings alone, independently of facts.
  - It's not synthetic.

Quine's criticism

- Some philosophers, most notably W. V. Quine, have attacked the distinction between analytic and synthetic statements.
- If you are interested in this, I recommend reading Carnap's reply to Quine in *The Philosophy of Rudolf Carnap*, ed. Paul Arthur Schilpp, Open Court 1963, pp. 915–922.

Logical probability in Carnap's sense

- An elementary probability sentence is a sentence which asserts that a specific hypothesis has a specific numeric probability.
  - This is an elementary probability sentence: "The probability of H given E is 0.51" (where H and E are specific sentences).
  - This is not: "The probability of H given E is equal to the proportion of babies that are boys."
- A probability concept is logical in Carnap's sense if all elementary sentences for it are logically determinate.

Inductive probability is logical in Carnap's sense

- Since inductive probability isn't degree of belief, the truth value of an elementary statement of inductive probability doesn't depend on some person's psychological state.
- It also doesn't depend on facts about the external world. In the coin example, the inductive probability that the coin will land heads is fixed by the evidence; it doesn't depend on the facts about the coin.
- Therefore, the truth value of an elementary statement of inductive probability doesn't depend on empirical facts at all.
- Therefore, inductive probability is logical in Carnap's sense.

Physical probability isn't logical in Carnap's sense

E.g., the physical probability that a coin will land heads depends on facts about the coin.

Inductive probability isn't the only probability concept that is logical in Carnap's sense.

- Let p be a probability function whose values are specified by definition.
- The elementary probability sentences for p are true or false in virtue of the definition of p; so they are logically determinate.
- So p is logical in Carnap's sense, though functions of this sort can't all be equal to inductive probability.

Definitions

- Inductive probability is the probability concept *of ordinary language* that is logical in Carnap's sense.
- Physical probability is the probability concept *of ordinary language* that isn't logical in Carnap's sense.

Questions

8. What does it mean for a probability concept to be logical in Carnap's sense? Explain the meaning of any technical terms that you use.
9. Is inductive probability logical in Carnap's sense? Justify your answer.
10. Is inductive probability the same thing as logical probability in Carnap's sense? Justify your answer.

Reference

Patrick Maher, "The Concept of Inductive Probability," *Erkenntnis* 65 (2006), 185–206.

- The magenta text is a link. You can get the article free with a uiuc internet connection.
- This lecture is based on sections 1 and 2 of the article.