Physical Probability 2
Physical Probability as a Theoretical Term

Patrick Maher
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Introduction

- We've seen that the frequency theory is an unsatisfactory attempt to explicate pp.
- An alternative is a *propensity theory*: pp is a propensity or tendency of the experiment to give particular outcomes; frequencies are evidence of this propensity but don't define it.
  - E.g., tossing a normal coin has a propensity of $1/2$ to give heads.
- *Objection:* "propensity" and "tendency" are too vague.
- *Solution:* Introduce an explicatum q for pp by stating postulates that relate q to p (an explicatum for inductive probability).
  - Since the meaning of q is given by these postulates, q is a T-term.

Principle of direct inference

Terminology

- A *Q-proposition* is a consistent conjunction of propositions of the form $q_X(O) = r$.
- "*Xa*" means a is a token of X.
- "*Oa*" means a has outcome O.

In what follows, Q is always a Q-proposition and a is a token event.

Principle of direct inference (DI)

If Q implies that $q_X(O) = r$, then $p(Oa|Xa.Q) = r$.

Since p is logical in Carnap's sense, DI is analytic; it is part of the definition of p.

Proof that q obeys the laws of probability

From DI, and the fact that p obeys the laws of probability, it follows that q_X also obeys the laws of probability. Proof:

Law 1: $q_X(O) \geq 0$

Let Q state the true value of $q_X(O)$. Then:

$$\begin{aligned} q_X(O) &= p(Oa|Xa.Q), \text{ by DI} \\ &\geq 0, \text{ since } p \text{ satisfies the laws of probability.} \end{aligned}$$

Law 2: If O is logically necessary, $q_X(O) = 1$

Suppose O is logically necessary. Let Q state the true value of $q_X(O)$. Then:

$$\begin{aligned} q_X(O) &= p(Oa|Xa.Q), \text{ by DI} \\ &= 1, \text{ since } p \text{ satisfies the laws of probability.} \end{aligned}$$

Law 3: If O_1 and O_2 are incompatible,

$$q_X(O_1 \vee O_2) = q_X(O_1) + q_X(O_2)$$

Suppose O_1 and O_2 are incompatible. Let Q state the true values of $q_X(O_1)$, $q_X(O_2)$, and $q_X(O_1 \vee O_2)$. Then:

$$\begin{aligned} q_X(O_1 \vee O_2) &= p(O_1 a \vee O_2 a | Xa.Q), \text{ by DI} \\ &= p(O_1 a | Xa.Q) + p(O_2 a | Xa.Q), \\ &\quad \text{since } p \text{ satisfies the laws of probability} \\ &= q_X(O_1) + q_X(O_2), \text{ by DI.} \end{aligned}$$

Principle of irrelevance

- The mere fact that an experiment is performed is not itself evidence regarding the pp of any outcome of that experiment.
- E.g., that a coin is tossed does not by itself give any evidence about the pp of the coin landing heads when tossed.

For any A, let $p(A)$ be an abbreviation for $p(A|T)$, where T is a tautology. The above observation motivates adopting:

Principle of irrelevance (IR)

$p(Q|Xa) = p(Q)$.

This is part of the definition of p and hence is analytic.

Learning the values of q

No analytic principle specifies the values of $q_X(O)$ for contingent O. Thus q isn't a logical concept and we need to learn its values empirically.

How this is done

1. Define a priori values $p(Q)$ for Q-propositions.
2. Use DI, IR, and Bayes's theorem to determine $p(Q|E)$ for evidence E.

Example

Let X = a particular die is tossed; O = it comes up six. Let Q_f (fair) mean that $q_X(O) = 1/6$ and Q_b (biased) mean that $q_X(O) = 1/2$. Suppose $p(Q_f) = p(Q_b) = 1/2$. What is $p(Q_b|Xa.Oa)$?

$$\begin{aligned}
p(Q_b|Xa.Oa) &= \frac{p(Oa|Xa.Q_b)p(Q_b|Xa)}{p(Oa|Xa.Q_b)p(Q_b|Xa) + p(Oa|Xa.Q_f)p(Q_f|Xa)}, \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{by Bayes's theorem} \\
&= \frac{(1/2)p(Q_b|Xa)}{(1/2)p(Q_b|Xa) + (1/6)p(Q_f|Xa)}, \text{ by DI} \\
&= \frac{(1/2)(1/2)}{(1/2)(1/2) + (1/6)(1/2)}, \text{ by IR} \\
&= 3/4 > p(Q_b).
\end{aligned}$$

Independence and the frequency theory

- I think there should be a further principle which says that, when q_X exists, the outcomes of repeated tokens of X are independent.
  - E.g., if X is tossing a particular coin, X^2 is tossing it twice, and H is the coin landing heads, $q_{X^2}(HH) = [q_X(H)]^2$.
- It can then be proved that, when q_X exists, any possible outcome O of X will probably occur randomly, and the relative frequency of O will probably be close to $q_X(O)$.
- These probabilities approach 1 as the number of repetitions approach infinity.
- This is like what the frequency theory says, except that probability 1 doesn't mean it must happen.

Existence of physical probabilities

It is widely agreed that pp's don't always exist. According to the explication given here:

- Pp is a relation between an experiment type and an outcome type. Other things don't have pp's.
  - E.g., there is not a pp of a scientific theory being true, because (i) this is a proposition, not an outcome type, and (ii) no experiment type is indicated.
- If X and O are experiment and outcome types (respectively), it's an empirical question whether there's a pp of X having outcome O.
  - If the occurrences of O don't appear random, or don't appear to be approaching a limit, that is evidence that there isn't a pp of X having outcome O.
  - E.g., if coin tosses came out HTHTHTHTHT..., that would be evidence that the independence principle isn't satisfied, and hence there isn't a pp of H or T on a toss of this coin.

Questions

1. What is a Q-proposition? What is the principle of direct inference? What is the principle of irrelevance?
2. State Maher's proof of the following:
   (a) $q_X(O) \geq 0$.
   (b) If O is logically necessary, $q_X(O) = 1$.
   (c) If O_1 and O_2 are incompatible, $q_X(O_1 \vee O_2) = q_X(O_1) + q_X(O_2)$.
3. According to Maher's explication, how do we learn the values of physical probabilities? (A brief verbal answer is sufficient.)
4. Do you think there is a physical probability that the next U.S. president will be a Democrat? Justify your answer using Maher's account of physical probability.