Law of Likelihood 1
The Law and Its Justification

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Favoring

People often say that evidence *favors* one hypothesis over another.

Examples

- The evidence that a patient tested positive for a disease favors the patient having the disease over the patient not having it.
- Getting 10 sixes in 30 tosses of a die is evidence that favors the die being biased for six over it being fair.
- Darwin claimed that evidence about fossils, etc., favored the hypothesis that species formed by natural selection over the hypothesis that they were independently created.

- In statements of this sort, it is always assumed that the hypotheses are incompatible.
- All these statements are relative to background evidence. For simplicity, I will omit mention of this, as we usually do.

Explication of favoring

Let p be an explicatum for inductive probability. The following are possible explicata for "E favors H_1 over H_2." ($H_{1\vee 2}$ is short for $H_1 \vee H_2$.)

1. $$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)}$$
   (E increases the ratio $p(H_1)/p(H_2)$)
2. $p(H_1|H_{1\vee 2}.E) > p(H_1|H_{1\vee 2})$
   (E incrementally confirms H_1 given $H_{1\vee 2}$)
3. $p(H_2|H_{1\vee 2}.E) < p(H_2|H_{1\vee 2})$
   (E incrementally disconfirms H_2 given $H_{1\vee 2}$)

These are equivalent if H_1 and H_2 are incompatible (proof at end).

To explicitly represent background evidence, put "D" on the right of "|" in every probability expression. (If there is no "|", add one.)

The law of likelihood

The law

Let H_1 and H_2 be incompatible hypotheses. Evidence E favors H_1 over H_2 iff the inductive probability of E is higher given H_1 than given H_2.

Corresponding explicatum statement

Let H_1 and H_2 be incompatible hypotheses. Then

$$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)} \text{ iff } p(E|H_1) > p(E|H_2).$$

The explicatum statement is demonstrably true (proof at end).
That is good reason to think the law is true.

Alleged counterexamples to the law of likelihood from "Likelihoodism, Bayesianism, and Relational Confirmation," by Branden Fitelson, *Synthese* (forthcoming):

Leeds' example (Fitelson p. 4)

- E = an ace has just been drawn from a deck of cards.
  H_1 = the card is the ace of hearts.
  H_2 = the card is the ace of spades or the ace of clubs.
- $ip(E|H_1) = ip(E|H_2) = 1$. So according to the law of likelihood, E doesn't favor either hypothesis over the other.
- But $ip(H_1|E) = 1/4$ and $ip(H_2|E) = 1/2$.
  Leeds and Sober think this shows that E favors H_2 over H_1.

My response: The values of posterior probabilities depend on two things, the prior probabilities and the evidence. In this case, the posterior probabilities are in the same ratio as the prior probabilities, so E doesn't favor H_1 over H_2.

Fitelson's example (p. 5)

This time, E = the card is a spade, H_1 = the card is the ace of spades, and H_2 = the card is black. In this example (assuming the standard probability model of card draws), $Pr(E|H_1) = 1 > Pr(E|H_2) = 1/2$, but it seems absurd to claim that E favors H_1 over H_2, as is implied by the [law of likelihood]. After all, E guarantees the truth of H_2, but E provides only nonconclusive evidence for the truth of H_1.

My response: Here H_1 and H_2 are compatible; in fact, H_1 entails H_2. The law of likelihood only applies to incompatible hypotheses (because the concept of favoring one hypothesis over another only makes sense for incompatible hypotheses).

Questions

1. State three possible explicata for "E favors H_1 over H_2." What is the logical relation between them?
2. State the law of likelihood and a corresponding explicatum statement. Explain briefly how one of these can be used to argue that the other is correct.
3. According to the law of likelihood, which (if either) of H_1 and H_2 is favored by E in the following cases? Justify your answers.
   (a) E = a ball drawn randomly from an urn and is black; H_1 = 10% of the balls in the urn are black, H_2 = 20% of them are black.
   (b) E = a die toss came up even; H_1 = it didn't come up 1; H_2 = it came up 2.
   (c) E = a die toss came up even; H_1 = it came up 4 or 6; H_2 = it came up 2.

Proofs (not on exam)

Equivalence of the explicata for favoring

Assume H_1 and H_2 are incompatible. Starting with (1), and using "$\Leftrightarrow$" for "iff," we have:

$$
\begin{aligned}
&p(H_1|E)/p(H_2|E) > p(H_1)/p(H_2)\\
&\quad\Leftrightarrow\quad p(E|H_1) > p(E|H_2), \text{ by Bayes's theorem}\\
&\quad\Leftrightarrow\quad p(E|H_1)p(H_2|H_{1\vee 2}) > p(E|H_2)p(H_2|H_{1\vee 2})\\
&\quad\Leftrightarrow\quad p(E|H_1)[1 - p(H_1|H_{1\vee 2})] > p(E|H_2)p(H_2|H_{1\vee 2})\\
&\quad\Leftrightarrow\quad p(E|H_1) > p(E|H_1)p(H_1|H_{1\vee 2}) + p(E|H_2)p(H_2|H_{1\vee 2})\\
&\quad\Leftrightarrow\quad \frac{p(E|H_1)p(H_1|H_{1\vee 2})}{p(E|H_1)p(H_1|H_{1\vee 2}) + p(E|H_2)p(H_2|H_{1\vee 2})} > p(H_1|H_{1\vee 2})\\
&\quad\Leftrightarrow\quad p(H_1|H_{1\vee 2}.E) > p(H_1|H_{1\vee 2}), \text{ by Bayes's theorem; this is (2)}\\
&\quad\Leftrightarrow\quad 1 - p(H_2|H_{1\vee 2}.E) > 1 - p(H_2|H_{1\vee 2})\\
&\quad\Leftrightarrow\quad p(H_2|H_{1\vee 2}.E) < p(H_2|H_{1\vee 2}); \text{ this is (3).}
\end{aligned}
$$

Explicatum statement corresponding to law of likelihood

For $i = 1, 2$ we have (by a simplified version of Bayes's theorem):

$$p(H_i|E) = \frac{p(E|H_i)p(H_i)}{p(E)}.$$

Dividing the expression for $i = 1$ by the one for $i = 2$ gives:

$$\frac{p(H_1|E)}{p(H_2|E)} = \frac{p(E|H_1)}{p(E|H_2)} \frac{p(H_1)}{p(H_2)}.$$

It follows that

$$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)} \text{ iff } \frac{p(E|H_1)}{p(E|H_2)} > 1.$$

Hence

$$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)} \text{ iff } p(E|H_1) > p(E|H_2).$$