Confirmation 2
Probability

(pp. 4–8)

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Philosophy 471

Fall 2006

Inductive and physical probability

Coin example

You are told that a coin is either two-headed or two-tailed but you have no information about which it is. The coin is about to be tossed. What is the probability that it will land heads? There are two natural answers: (i) 1/2; (ii) either 0 or 1 but I don't know which. These different answers correspond to different meanings that the word "probability" has in ordinary language.

Terminology

- Inductive probability: The meaning of "probability" in which (i) is the natural answer.
- Physical probability: The meaning of "probability" in which (ii) is the natural answer.

Physical probability depends on empirical facts (e.g., the nature of the coin). Inductive probability doesn't.

Theories of physical probability

- *Frequency theory:* The physical probability of an event is the relative frequency with which it happens in the long run.
- *Propensity theory:* The physical probability of an event is the propensity that the circumstances have to produce that event.

Confirmation isn't about physical probability

- Physical probabilities seem not to exist in many contexts in which we talk about confirmation.
  - E.g., scientific theories can be confirmed but don't seem to have a physical probability of being true.
- Physical probabilities depend on empirical facts, but whether E confirms H depends only on E and H.

Neither of these things is true of inductive probability.

Subjective probability

The *subjective probability* of H for person X at time t is the degree to which X believes H at time t.

Inductive probability $\neq$ subjective probability

Suppose I claim that the available evidence makes it probable that scientific theory H is true.

- This is a statement of inductive probability.
- I can't prove my claim is true by proving that I have a high degree of belief in H.
- If inductive probability were subjective probability, I could prove my claim that way.
- So inductive probability is not subjective probability.

Logical probability

The term "logical probability" is often used without a clear meaning. We'll define a precise sense based on work of Rudolf Carnap in his *Logical Foundations of Probability*.

Definitions

- An elementary probability sentence is a sentence which asserts that a specific hypothesis has a specific probability.
  - This is an elementary probability sentence: "The probability of H given E is 0.51" (where H and E are specific sentences).
  - This is not: "The probability of H given E is equal to the proportion of babies that are boys."
- A logically determinate sentence is a sentence whose truth or falsity is determined by meanings alone, independently of empirical facts.
- A probability concept is logical in Carnap's sense if all elementary sentences for it are logically determinate.

Inductive probability is logical in Carnap's sense

- Since inductive probability isn't degree of belief, the truth value of an elementary statement of inductive probability doesn't depend on some person's psychological state.
- It also doesn't depend on facts about the external world. In the coin example, the inductive probability that the coin will land heads is fixed by the evidence; it doesn't depend on the facts about the coin.
- Therefore, the truth value of an elementary statement of inductive probability doesn't depend on empirical facts at all.
- Therefore, inductive probability is logical in Carnap's sense.

Questions

1. Describe a situation in which the inductive probability of a die landing six is (a) the same as its physical probability; (b) different to its physical probability.
2. Is inductive probability the same thing as subjective probability? Justify your answer.
3. What does it mean for a probability concept to be logical in Carnap's sense? Is inductive probability logical in Carnap's sense? Justify your answer to the latter question.

Existence of inductive probabilities

It has often been claimed that logical probabilities don't exist. If that is right, inductive probabilities don't exist or aren't logical.

Ramsey's argument (1926)

1. There is very little agreement on the values of probabilities in the simplest cases.

   E.g., nobody can state a numerical value for the probability of one thing being red given that another thing is red.

2. The simplest cases are the ones where logical relations should be most clear.
3. Therefore, logical probabilities don't exist.

This argument continues to be cited approvingly (Gillies 2000; Hacking 2001).

Critique of Ramsey's argument as applied to IP

- The example Ramsey cites (nobody can state a numerical value) shows *agreement*, not disagreement. It just illustrates that inductive probabilities often lack numeric values.
- There are also simple cases where numeric values for inductive probabilities are widely agreed on.

  E.g., the probability that a ball is white, given only that it is either white or black, is 1/2.
- Inductive probabilities needn't be clearest in the simplest cases.
  - The concept of inductive probability is learned largely from examples of its application in ordinary life.
  - Many of these examples are complex.
  - Hence, its application may sometimes be clearer in realistic complex situations than in simple situations that never arise in ordinary life.

So both premises of Ramsey's argument are false, and the first for two reasons!

Argument from the paradoxes of indifference

1. Logical probabilities must be determined by a general rule.
2. The only rule that has been proposed is the *Principle of Indifference:* If evidence doesn't favor either H_1 or H_2 over the other then H_1 and H_2 are equally probable.
3. This rule leads to contradictions. Example:
    - Suppose we are given that a cube has sides between 1 and 2 inches long.
    - The Principle of Indifference implies that the probability its sides are between 1 and 1.5 inches is 1/2.
    - But the volume is between 1 and 8 cubic inches. So the Principle of Indifference implies that the probability its volume is between 1 and 4 cubic inches is 1/2.
    - These results are inconsistent. ($1.5^3 = 3.375$, not 4.)
4. So logical probabilities don't exist.

Many authors give some version of this argument, e.g., Gillies 2000.

Critique of this argument as applied to IP

Inductive probabilities don't need to be determined by a general rule.

- We learn the concept of inductive probability from examples, not by being given a general rule. (Most concepts of ordinary language are like this.)
- Hence we can have knowledge of particular inductive probabilities without being able to state a general rule that covers these cases.

Thus the first premise of the argument is false.

Argument that inductive probabilities exist

Let WB = “The probability that a ball is white, given only that it is either white or black, is 1/2.”

1. Most people think WB is true.
2. WB is a statement of inductive probability, hence is logically determinate. It is also simple.
3. Since people normally use their language correctly, it’s unlikely most people are wrong about a simple logically determinate sentence.
4. So it’s unlikely WB is false.
5. So, since WB implies that an inductive probability exists, there’s good reason to think an inductive probability exists.

Likewise in many other cases.

Questions

4. Are the following arguments sound as applied to inductive probabilities? Justify your answer.
   (a) There is very little agreement on the values of probabilities in the simplest cases. But the simplest cases are the ones where logical relations should be most clear. Therefore, logical probabilities don't exist.
   (b) Logical probabilities must be determined by a general rule. But the only rule that has been proposed is the Principle of Indifference and it leads to contradictions. Therefore, logical probabilities don't exist.
5. What is Maher's argument that inductive probabilities exist?