# Lecture 9
# Maher on Inductive Probability

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# Two concepts of probability

**Example**

You know that a coin is either two-headed or two-tailed but you have no information about which it is. The coin is about to be tossed. What is the probability that it will land heads? There are two natural answers:

- $1/2$
- Either 0 or 1 but I don't know which.

So "probability" in ordinary language is ambiguous.

**Terminology**

- Inductive probability: The meaning of "probability" in which "$1/2$" is the natural answer.
- Physical probability: The meaning of "probability" in which "0 or 1" is the natural answer.

Abbreviations: ip = inductive probability, pp = physical probability.

# Not degree of belief

## Arguments that ip $\neq$ degree of belief

1. Dictionaries don't give degree of belief as a meaning of "probability." Also, if ordinary people are asked what "probability" means, they won't say it means a person's degree of belief. So degree of belief isn't a meaning of "probability" in ordinary language. But ip is.
2. If ip is degree of belief then, when people make assertions about ip, they are presumably making assertions about their own degrees of belief. Then statements of ip by different people can't contradict one another. But they can.
3. If ip is degree of belief then claims about ip can be justified by evidence that the speaker has the relevant degrees of belief. But claims about ip can't be justified this way.

## Expression vs assertion

Some statements in the literature suggest this argument:

> *Our assertions about inductive probabilities express our degrees of belief, so these assertions can only mean that we have these degrees of belief.*

That argument is invalid.

- All sincere intentional assertions *express* our beliefs but most such assertions are not *about* our beliefs. We need to distinguish between the content of an assertion and the state of mind which that assertion expresses.
- Analogy: If I say it's raining, I'm expressing my belief that it's raining but I'm not asserting that I have this belief; I'm asserting that it's raining.

# Form of ip statements

## Arguments of ip

- Every ip is a probability *of* some proposition $H$ *given* some proposition $E$. Changing either $H$ or $E$ can change the value of the ip. *Notation:* $ip(H|E)$ = the ip of $H$ given $E$.

  *Example:* $H$ = the coin lands heads, $E$ = the coin is either two-headed or two-tailed and is about to be tossed, $E'$ = the coin has a head on one side. $ip(H|E) = 1/2$; $ip(H|E\&E') = 1$.

- It is customary to call $H$ the *hypothesis* and $E$ the *evidence*. However, $H$ and $E$ can be any propositions whatever.
- In ordinary language the evidence often isn't stated. Then the evidence is usually the evidence possessed by the speaker or the scientific community.

  *Example:* "Humans probably evolved in Africa" means "the probability that humans evolved in Africa, given the evidence now available, is high."

## Values of ip

- Some ips have a numeric value.

  *Example:* With $H$ and $E$ as before, $ip(H|E) = 1/2$.

- Many ips don't have a numeric value.

  *Example:* $ip$(humans evolved in Africa|my evidence) is high but doesn't have a numeric value.

- When an ip lacks a numeric value we may still be able to express inequalities regarding it.

  *Example:* $ip$(humans evolved in Africa|my evidence) $> 1/2$.

# Logical probability

**Definition**

An elementary sentence for a function is a sentence that says the function has a specific value for specific arguments.

*Example an elementary sentence for ip:* "The ip that a coin landed heads, given that it either landed heads or tails, is 1/2."

*Not elementary:* "The ip of my favorite proposition, given my evidence, equals the proportion of Chicago residents with red hair."

**Definition**

A function is logical if all true elementary sentences for it are analytic.

## Argument that ip is logical

1. The truth of elementary sentences of ip doesn't depend on the speaker's psychological state, since ip isn't degree of belief.
2. It also doesn't depend on external facts, as pp does. (In the coin example, the ip is $1/2$ regardless of whether the coin is 2-headed or 2-tailed.)
3. Thus, it doesn't depend on any empirical facts at all.
4. Hence, true elementary sentences of ip are analytic.
5. Therefore, ip is logical.

# Questions

1. Describe a situation in which the inductive probability of a die landing six, given the available evidence, is (a) the same as its physical probability, (b) different to its physical probability.
2. "Probability measures the confidence that a particular individual has in the truth of a particular proposition" (Leonard J. Savage, *The Foundations of Statistics*, p. 3). Is this a correct account of inductive probability? Justify your answer.
3. Give an example of an elementary sentence for inductive probability in which the hypothesis is that it will rain tomorrow.
4. What does it mean for a function to be logical? Is inductive probability logical? Justify your answer to the latter question.

# Reference

Patrick Maher.
The concept of inductive probability.
*Erkenntnis*, 65:185–206, 2006.
Online in SpringerLink (free with a uiuc connection).