Lecture 5
Laplace on Probability

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Pierre-Simon Laplace

1749: Born in Normandy.

1774: First publication on probability.

1789: French revolution.

1804: Napoleon became emperor.

1806?: This portrait painted.

1812: *Analytic Theory of Probabilities*, 1st edition.

1814: Monarchy restored.

1827: Died in Paris.

Task of probability theory

- Laplace said that in probability theory we are given the probabilities of some simple events and the task is to determine the probabilities of compound events.
- He called the probabilities of the simple events *possibilities*.
- *In the analysis of chances, one aims to find the probabilities of events composed, according to a given law, of simple events with given possibilities.* (384)

Example (by me)

- *Simple event:* a coin lands heads on a toss.
- *Possibility of this simple event:* $1/2$ (suppose).
- *Compound event:* At least one head in two tosses.
- *Probability of the compound event:* $1/4 + 1/4 + 1/4 = 3/4$.

How possibilities are known (384–85)

These possibilities may be determined in the following three ways:

1. *a priori, when from the nature of the events one sees that they are possible in a given ratio; for example, in tossing a coin, if the coin is homogeneous and its two faces are entirely alike, we judge that heads and tails are equally possible;*
2. *a posteriori, by making many repetitions of the experiment that can produce the event in question and seeing how often the event occurs;*
3. *finally, by considering the reasons which may determine us to pronounce on the existence of the event; for example, if the skills of two players A and B are unknown, since we have no reason to suppose A stronger than B, we conclude that the probability of A winning a game is 1/2.*

Another example (by me)

Suppose we want to know the possibility that a die will come up six when tossed. The three methods:

1. We know the die has six sides and is symmetric (not biased) and conclude that the possibility is $1/6$.
2. We toss the die 1000 times, it comes up six 173 times, and we conclude that the possibility is approximately 0.173.
3. We know nothing about the die except that it has six sides numbered 1 to 6, so we have no reason to believe one side will come up rather than another, and we conclude that the possibility is $1/6$.

Two kinds of possibility (385)

The first method gives the absolute possibility of the events; the second gives approximate knowledge of it, . . . and the third gives only their possibility relative to our knowledge.

Compatibility with determinism (385)

Every event being determined by the general laws of the universe, there is only probability relative to us and, for this reason, the distinction between absolute and relative possibility may appear imaginary. But one must observe that, among the circumstances that concur in the production of events, some change at every moment, such as the movement that the hand imparts to dice, and it is the union of these circumstances that one calls chance. There are others that are constant, such as the ability of the players, the tendency of the dice to fall on one face rather than the others, etc.; these form the absolute possibility of events and knowledge of them that is more or less incomplete forms their relative possibility.

- Absolute possibility isn't 0 or 1 because it is only relative to constant causes, not all causes.
- Relative possibility can differ from absolute possibility because we don't always know all the constant causes.

The distinction has been overlooked (385–86)

The work done up to now in the theory of chance assumes knowledge of the absolute possibility of events and, with the exception of some remarks that I have given in [earlier papers], I do not know that anyone has considered the case in which only their relative possibility is known. This case contains many interesting questions and is relevant to most problems concerning games. The reason mathematicians have not paid particular attention to this is presumably that they thought the same methods applied to it as to the case where the absolute possibility of the events is known. However, the essential difference between these possibilities can significantly alter the results of calculations, so that one is often exposed to considerable errors if one employs them in the same manner.

Notation (not in Laplace): $p(A)$ = the probability of A.

Why it matters

Given absolute possibilities, $p(A \& B) = p(A)p(B)$.
Given relative probabilities, this isn't true in general.

Modern terminology: Absolute probabilities are *independent*.

Example of coin tossed twice (Laplace 1825, ch. 7)

H_i = coin lands heads on the ith toss.

1. Coin known to be fair.
   - Absolute possibility of $H_i = 1/2$.
   - $p(H_1 \& H_2) = p(H_1)p(H_2) = (1/2)(1/2) = 1/4$.
2. Coin known to be biased but direction of bias unknown.
   - Relative possibility of $H_i = 1/2$.
   - $p(H_1 \& H_2) = p(H_1)p(H_2|H_1) = (1/2)\, p(H_2|H_1)$.
     $p(H_2|H_1) > 1/2$ because heads on the first toss is evidence the bias is towards heads. So $p(H_1 \& H_2) > 1/4$.

Quantitative analysis (Laplace 1825, ch. 7)

Suppose the bias alters the (absolute) possibility by 1/20 but the direction of this bias is unknown. Then the absolute probability of two heads is

$$\text{either } \left(\frac{11}{20}\right)^2 \text{ or } \left(\frac{9}{20}\right)^2.$$

We have no reason to favor one over the other, so the relative probability of 2 heads is the average of these, i.e.,

$$\frac{1}{2}\left[\left(\frac{11}{20}\right)^2 + \left(\frac{9}{20}\right)^2\right] = \frac{101}{400} > \frac{1}{4}.$$

Laplace on relativity to us

- *Every event being determined by the general laws of the universe, there is only probability relative to us.* (385)
- *Probability is relative, in part to [our] ignorance, in part to our knowledge.* (1825, ch. 2)

Criticism (by me)

- Absolute probabilities aren't relative to what we know. They are relative to the "fixed causes" but we often don't know what those are.
- *Example:* If we know a coin is biased but not the direction of the bias, the probability it will land heads, relative to what we know, is $1/2$, but the absolute probability isn't $1/2$.
- *In general:* When relative prob $\neq$ absolute prob, they can't both be relative to what we know.

Questions

1. How would Laplace determine the possibility of a coin landing heads (a) a priori, (b) a posteriori, and (c) by considering reasons we have? What kind of possibility is determined by each method?
2. Why are absolute possibilities not always 0 or 1, according to Laplace? Why do relative possibilities often differ from absolute possibilities?
3. A die will be tossed twice. What is the probability that it will land six each time given (a) the die is fair, (b) the die is biased but the direction of the bias is unknown? Justify your answers.
4. Laplace said "there is only probability relative to us." Is that correct? Justify your answer.

References

Pierre-Simon Laplace.
Mémoire sur les probabilités, 1781.
Reprinted in Laplace's *Oeuvres Completes* vol. 9, 383–485.
On Google books. Numbers in parentheses are page numbers of this edition. Translations are mine.

Pierre-Simon Laplace.
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5th edition, 1825.
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