Lecture 34
Newton on Gravity

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Outline of Newton's *Principia*

- Definitions
- Axioms, or the Laws of Motion
- Book 1: The Motion of Bodies
- Book 2: The Motion of Bodies
- Book 3: The System of the World

Today we'll look at Book 3. Newton is here concerned with what we call the solar system (system of the sun).

Rules of reasoning in philosophy

Rule 1 (794)

No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena.

As the philosophers say: Nature does nothing in vain, and more causes are in vain when fewer suffice. For nature is simple and does not indulge in the luxury of superfluous causes.

Rule 2 (795)

Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same.

Examples are the cause of respiration in man and beast, or the falling of stones in Europe and America.

Rule 3 (795)

Those qualities of bodies that ... belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally.

Example: *That all bodies are movable and persevere in motion or in rest by means of certain forces (which we call forces of inertia) we infer from finding these properties in the bodies that we have seen.*

Rule 4 (796)

In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more accurate or liable to exceptions.

Phenomenon 6 (801)

The moon, by a radius drawn to the center of the earth, describes areas proportional to the times.
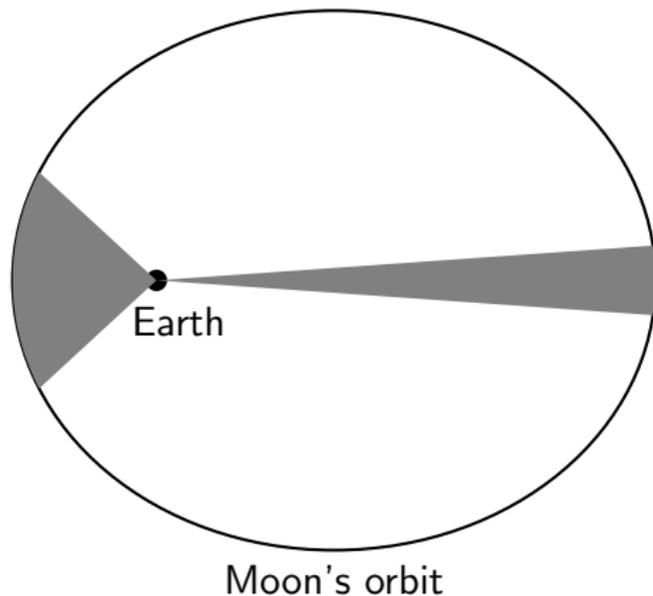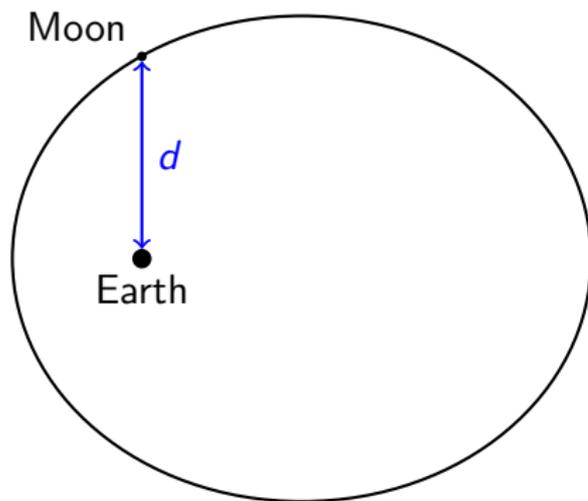

The shaded areas are equal and so are traced in equal times.

Earth

Moon's orbit

Proposition 3 (802)

The force by which the moon is maintained in its orbit is directed toward the earth and is inversely as the square of the distance of its places from the center of the earth.

The force holding the Moon in orbit is directed toward Earth and proportional to $1/d^2$.

Newton's argument for Proposition 3:

- That the force is toward Earth follows from Phenomenon 6: the line from Earth to Moon traces out areas proportional to the times. (I won't go through the proof.)
- That the force is proportional to $1/d^2$ follows from the Moon's apogee being stationary.
    - The Moon's apogee is the point where the Moon is farthest from Earth.
    - Newton proved that if the apogee is stationary then the force is proportional to $1/d^2$. (I won't go through the proof.)
    - He noted that the apogee isn't really stationary; it goes forward $3°3'$ per revolution of the Moon. But he said: *This motion of the apogee arises from the action of the sun (as will be pointed out below) and accordingly is to be ignored here.* (803)

The moon gravitates toward the earth and by the force of gravity is always drawn back from rectilinear motion and kept in its orbit.

Argument: We know how fast the Moon is accelerating towards Earth, by observation. We know the force causing that is proportional to $1/d^2$, by Prop. 3. Hence we can calculate what the acceleration of the Moon would be if we brought the Moon down to the surface of Earth.

$$\frac{a_2}{a_1} = \frac{F_2}{F_1}, \text{ by 2nd law of motion}$$
$$= \frac{d_1^2}{d_2^2}, \text{ by Prop. 3.}$$

We know one acceleration and both distances, so can calculate the other acceleration. (Continued on next slide.)

Argument for Proposition 4 (continued):

Newton did the calculation and found that, if the Moon were brought down to Earth, its acceleration would be the same as the acceleration with which bodies on Earth are observed to fall. The force that makes bodies on Earth fall is gravity (that's what "gravity" meant). So either:

- the force acting on the Moon and the force of gravity are different forces but produce the same acceleration at the Earth's surface; or
- the force acting on the Moon is gravity.

Rule 2 says we should take the second alternative.

Newton and the falling apple

Everyone knew apples fall due to gravity. Newton's new idea was that this force of gravity might extend all the way up to the moon and be what keeps the moon in orbit around the earth.
Proposition 4 says that idea is right.

1. According to Newton, what is the direction of the force that keeps the moon in its orbit? From what did Newton deduce this?

2. According to Newton, how is the force on the moon related to the moon's distance from the earth? From what did Newton deduce this?

3. Explain how Newton argued, from the properties of the force that keeps the moon in its orbit, that this force is gravity. Say where Newton's rules of reasoning are used in this argument.

Proposition 5 (805)

The [moons of Jupiter] gravitate toward Jupiter, the [moons of Saturn] gravitate toward Saturn, and the circumsolar planets gravitate toward the sun, and by the force of their gravity they are always drawn back from rectilinear motions and kept in their curvilinear orbits.

Argument:

- The moons of Jupiter are held in orbit by a force tending to the center of Jupiter and inversely proportional to the square of the distance. Similarly for the moons of Saturn and the circumsolar planets. (Props. 1, 2)
- Thus the force keeping these bodies in orbit is just like that keeping Earth's moon in orbit.
- So, by Rule 2, the force keeping these bodies in orbit is the same as the one keeping Earth's moon in orbit, i.e., gravity.

Therefore, there is gravity toward all planets universally. For no one doubts that Venus, Mercury, and the rest are bodies of the same kind as Jupiter and Saturn.

This uses Rule 3.

- Possessing gravity is a property that belongs "to all bodies on which experiments can be made," i.e., all planets that (so far as Newton knew) have moons orbiting them.
- So, by Rule 3, it belongs to "all planets universally."

4. Explain how, after arguing that the moon is kept in its orbit by gravity, Newton argued that all planets have gravity. Say where Newton's rules of reasoning are used in this argument.

Isaac Newton.
The Principia: Mathematical Principles of Natural Philosophy.
University of California Press, 1999.
Translation by I. Bernard Cohen and Anne Whitman.
(Numbers in parentheses are page numbers of this book.)