Lecture 28
Galileo's Experimental Evidence

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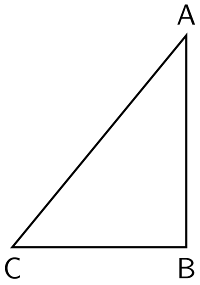
Scientific Thought I
Fall 2009

Introduction

- Galileo claimed that naturally accelerated motion is uniformly accelerated.
- *In this belief we are confirmed mainly by the consideration that experimental results are seen to agree with and exactly correspond with those properties which have been, one after another, demonstrated by us.* (160)
- In the previous lecture we saw some properties of uniform acceleration that Galileo demonstrated (Theorems 1 and 2; Corollary 1).
- Today we'll look at two more demonstrated properties (Theorems 3 and 4) and then at Galileo's experimental results.

Theorem 3 (185)

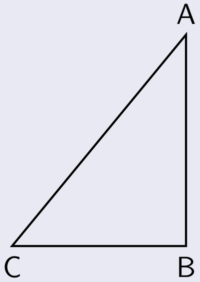
If one and the same body, starting from rest, falls along an inclined plane and also along a vertical, each having the same height, the times of descent will be to each other as the lengths of the inclined plane and the vertical.



For a body starting from rest at A, Theorem 3 says:

$$\frac{\text{time to traverse AC}}{\text{time to traverse AB}} = \frac{AC}{AB}.$$

Sagredo's proof of Theorem 3 (186)



1. Either way the initial speed is zero.
2. Either way the final speed is the same, by Galileo's assumption.
3. So the mean of the initial and final speeds is the same either way.
4. Theorem 1 (for uniformly accelerated motion) says the body covers the distance in the same time that it would if moving uniformly at the mean speed.
5. So we can treat the body as if moving with the same uniform speed either way.
6. So the time taken is proportional to the distance covered, by Theorem 1 for uniform motion. Q.E.D.

Theorem 4 (187)

The times of descent along planes of the same length but of different inclinations are to each other in the inverse ratio of the square roots of their heights.

Algebraic statement

For planes of the same length with heights h_1 and h_2,

$$\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}.$$

I won't go through Galileo's proof of this; I just note that:

- The proof of Theorem 4 uses Theorem 3.
- Since the proof of Theorem 3 used Galileo's assumption, the proof of Theorem 4 depends on that assumption.

Comparison of Theorems 2 and 4

Theorem 2: For planes at the same angle with different lengths,

$$\frac{t_1}{t_2} = \sqrt{\frac{s_1}{s_2}}.$$

Theorem 4: For planes at different angles with the same length,

$$\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}.$$

Galileo's experiments

Apparatus (178–179, Salviati speaking)

A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat.

A cubit is the length between a person's elbow and finger tip, or about 18 inches.

Results (179)

Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i.e., of the channel, along which we rolled the ball.

This confirms that Theorem 2 holds for naturally accelerated motion. E.g., when the ball is rolled 1/4 of the length:

$$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

More results (179)

We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them.

The reference here is to Theorem 4, so in these experiments:

$$\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}.$$

How time was measured (179)

For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results.

Evidence for Galileo's claim

Galileo claimed that naturally accelerated motion is uniformly accelerated. He said the main reason to believe this is that the consequences deduced from it agree with experiment. We've now seen that the consequences Galileo verified were:

Theorem 2: For planes at the same angle with different lengths,

$$\frac{t_1}{t_2} = \sqrt{\frac{s_1}{s_2}}.$$

Theorem 4: For planes at different angles with the same length,

$$\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}.$$

Evidence for Galileo's assumption

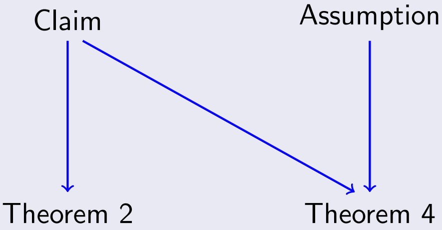
Galileo made the assumption that the speeds acquired in moving down planes of different inclination are equal when the heights of the planes are equal. He said the main reason to believe this is that the consequences deduced from it agree with experiment. We've now seen that Galileo verified one consequence deduced from it, namely:

Theorem 4: For planes at different angles with the same length,

$$\frac{t_1}{t_2} = \sqrt{\frac{h_2}{h_1}}.$$

The deduction of this also depended on his claim that naturally accelerated motion is uniformly accelerated.

Logical diagram



Arrows are deductions. Inference goes in the opposite direction.

Questions

1. Explain Sagredo's proof of the following theorem for uniformly accelerated motion:

   > *If one and the same body, starting from rest, falls along an inclined plane and also along a vertical, each having the same height, the times of descent will be to each other as the lengths of the inclined plane and the vertical.*

2. What was Galileo's experimental evidence for his claim that naturally accelerated motion is uniformly accelerated? Why is that evidence relevant?

3. What was Galileo's experimental evidence for the following assumption? Why is that evidence relevant?

   > *The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal.*

Reference

Galileo Galilei.
Dialogues Concerning Two New Sciences.
Macmillan, 1914.
Translated by Henry Crew and Alfonso de Salvio.
Online in facsimile pdf (16MB) and html.
Numbers in parentheses are page numbers of this edition.