Lecture 27
Galileo on Properties of Uniform Acceleration

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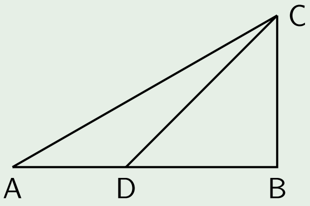
Galileo's assumption (169)

The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal.

Illustration by Salviati (169)



The Author calls the perpendicular CB the "height" of the planes CA and CD; he supposes that the speeds acquired by one and the same body, descending along the planes CA and CD to the terminal points A and D are equal since the heights of these planes are the same, CB; and also it must be understood that this speed is that which would be acquired by the same body falling from C to B.

Sagredo accepts the assumption

- Sagredo: *Your assumption appears to me so reasonable that it ought to be conceded without question, provided of course there are no chance or outside resistances, and that the planes are hard and smooth, and that the figure of the moving body is perfectly round, so that neither plane nor moving body is rough.* (170)
- Salviati: *Your words are very plausible; but I hope by experiment to increase the probability to an extent which shall be little short of a rigid demonstration.* (He goes on to describe experiments which show that an analogous result is true for weights attached to threads.)
- Sagredo: *The argument seems to me so conclusive and the experiment so well adapted to establish the hypothesis that we may, indeed, consider it as demonstrated.* (172)

Salviati dismisses the experiments

- *I do not wish, Sagredo, that we trouble ourselves too much about this matter, since we are going to apply this principle mainly in motions which occur on plane surfaces, and not upon curved, along which acceleration varies in a manner greatly different from that which we have assumed for planes.* (172)
- *Let us then, for the present, take this as a postulate, the absolute truth of which will be established when we find that the inferences from it correspond to and agree perfectly with experiment.* (172)

My interpretation: Galileo thinks the argument from weights on threads is suggestive, so he includes it. But he knows it doesn't strictly prove the assumption, so he doesn't rely on it.

Summary

Galileo's discussion suggests the following reasons to believe his assumption:

1. It is inherently plausible.
2. Experiment shows that a parallel phenomenon holds for weights on strings.
3. Most important: The consequences deduced from it agree with experiment.

Theorem 1 (173)

The time in which any space is traversed by a body starting from rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed and the speed just before the acceleration began.

Example

Two cars travel from A to B.

- One starts from rest at A and accelerates uniformly; when it arrives at B it is going 60 MPH.
- The other travels from A to B at a uniform speed of 30 MPH.

Theorem 1 says they take the same time to go from A to B.

Proof of Theorem 1 (simplified)

1. Let AB represent a period of time.
2. The horizontal distance between AE and AB represents the speed of a body that starts from rest and accelerates uniformly.
3. GF similarly represents a body that travels with the mean speed for the whole time AB.
4. The body traveling with the mean speed has a higher speed in the first half of the time but a slower speed in the second half, with the differences equal and opposite at corresponding instants.
5. Therefore, the two bodies will have traveled the same distance at the end of the time AB.

Questions

1. What reasons for believing the following assumption were suggested by Galileo? To which reason did he give the greatest weight?

   > *The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal.*

2. Explain Galileo's proof of the following theorem:

   > *The time in which any space is traversed by a body starting from rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed and the speed just before the acceleration began.*

Theorem 2 (174)

The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances.

Algebraic statement (not in Galileo)

If a body is accelerating uniformly from rest, and t_1 and t_2 are the times it takes to traverse distances s_1 and s_2 respectively, then

$$\frac{s_1}{s_2} = \frac{t_1^2}{t_2^2}.$$

Example

If a body accelerating uniformly from rest traverses one meter in the first second, then after two seconds it will have traversed four meters.

Algebraic proof of Theorem 2

Let s_i be the distance a body accelerating uniformly from rest travels in time t_i; let v_i be the velocity it then has.

$$\begin{aligned} s_i &= \text{distance traveled in } t_i \text{ by a body moving with} \\ &\quad \text{uniform speed } v_i/2 \text{ (by Theorem 1)} \\ &= \frac{v_i}{2} t_i \quad \text{(distance = velocity} \times \text{time for a body with uniform speed).} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{s_1}{s_2} &= \frac{v_1 t_1}{v_2 t_2} \\ &= \frac{t_1^2}{t_2^2} \quad \text{(the body is uniformly accelerated, so velocity is proportional to time).} \end{aligned}$$

Corollary 1 (175)

For a body accelerating uniformly, the spaces traversed in equal intervals of time, starting at the beginning of motion, are in the ratio of the odd numbers 1, 3, 5, 7, . . .

Example

If a body accelerating uniformly from rest goes 1 foot in the first second, then it goes 3 feet in the next second, 5 feet in the third second, 7 feet in the fourth second, and so on.

Why it is true

Time	1	2	3	4	. . .	
Total distance	1	4	9	16	. . .	(by Theorem 2)
Distance in time interval	1	3	5	7	. . .	

Questions

3. Explain Galileo's proof of the following theorem:

   > *The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances.*

4. Explain why the following is a corollary of a theorem of Galileo's:

   > *For a body accelerating uniformly, the spaces traversed in equal intervals of time, starting at the beginning of motion, are in the ratio of the odd numbers 1, 3, 5, 7, . . .*

Reference

Galileo Galilei.
Dialogues Concerning Two New Sciences.
Macmillan, 1914.
Translated by Henry Crew and Alfonso de Salvio.
Online in facsimile pdf (16MB) and html.
Numbers in parentheses are page numbers of this edition.