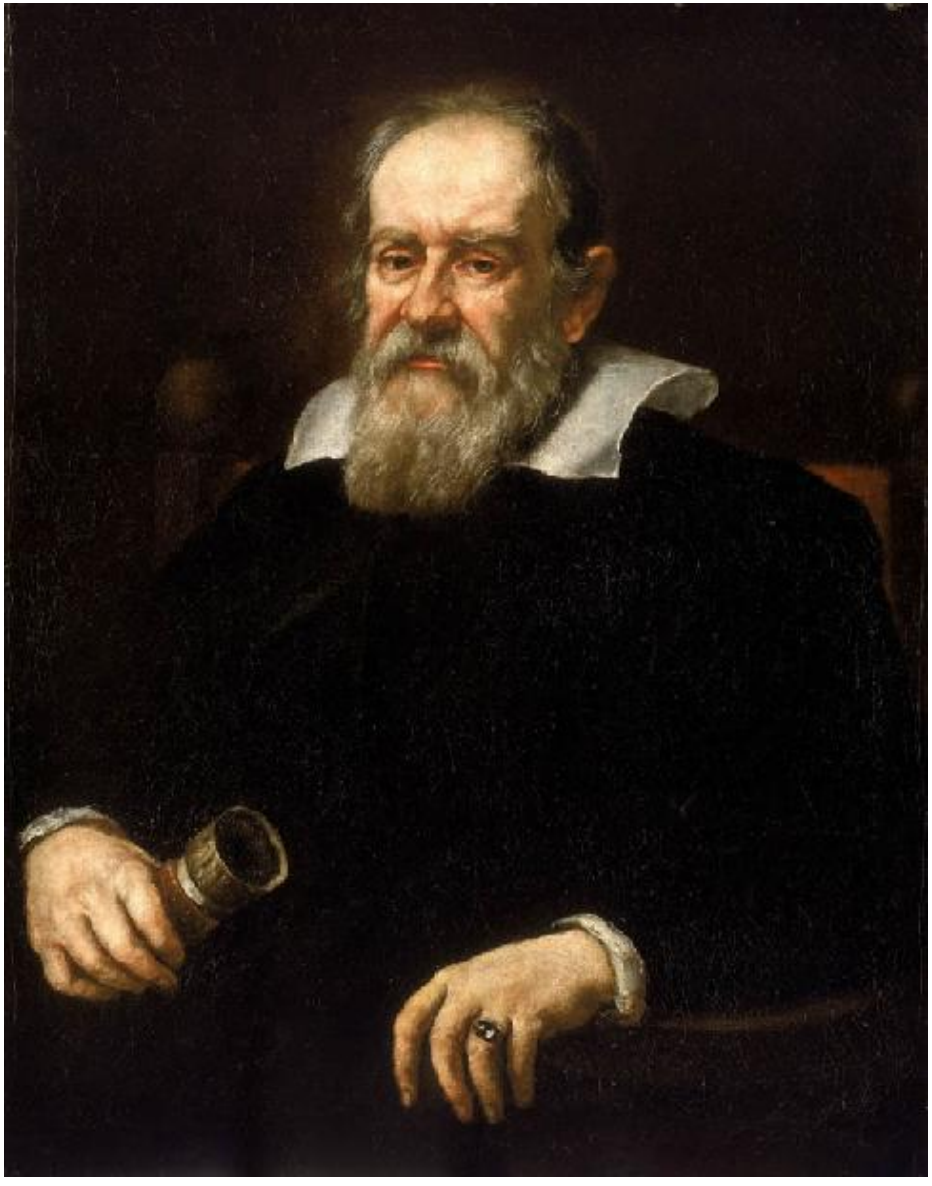
Lecture 25
Galileo on Uniform Motion

Patrick Maher

Scientific Thought I
Fall 2009

Galileo Galilei



1564: Born in Pisa, Italy.

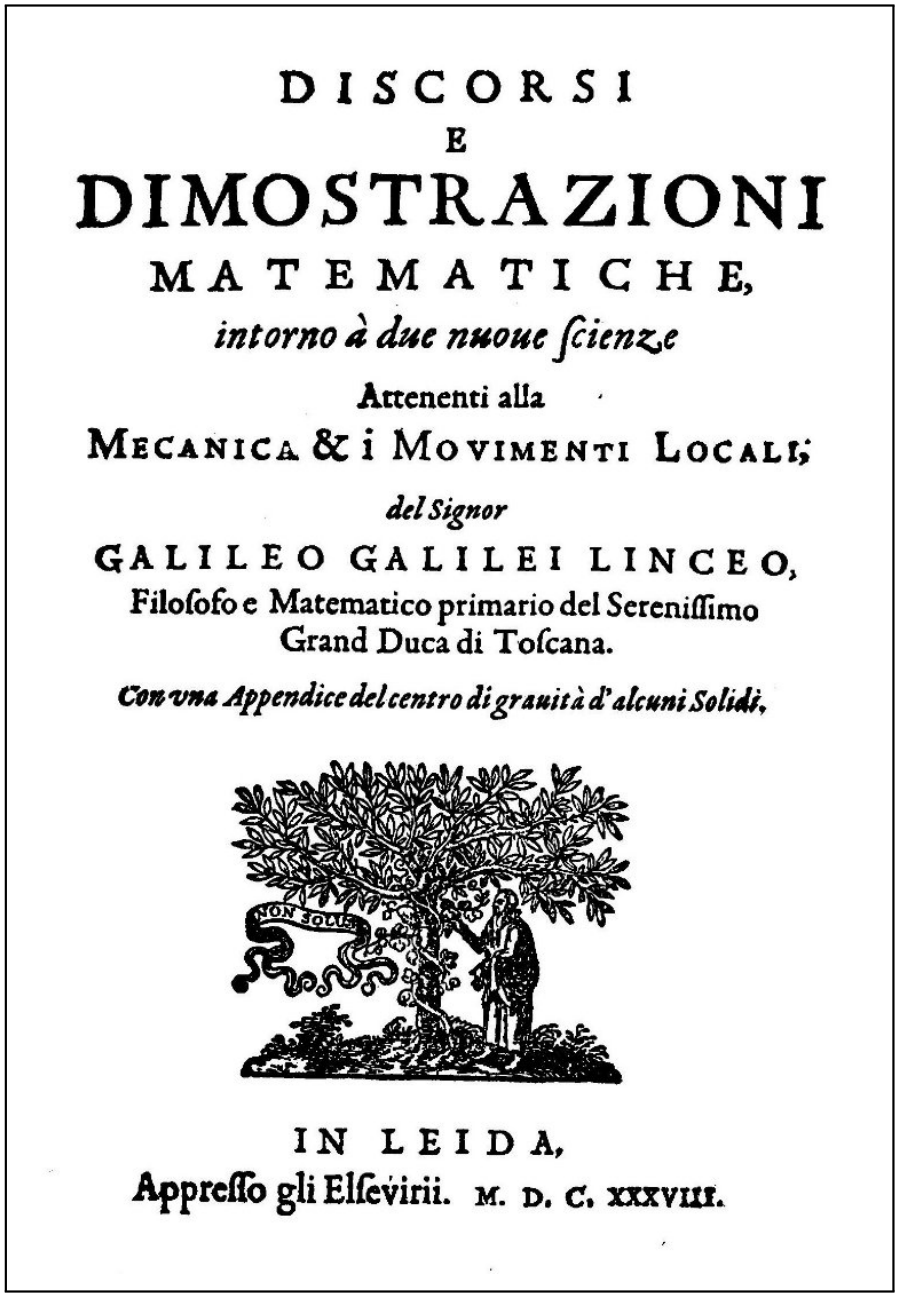
1632: Published a book defending Copernicus's theory. Tried by the Inquisition and sentenced to house arrest for the remainder of his life. Publication of any work by him was banned.

1638: His book *Two New Sciences* was published in the Netherlands.

About 1639: This portrait was painted.

1642: Died near Florence, Italy.

Two New Sciences



DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI;
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

- Three characters read some writings of Galileo (in Latin) and discuss them (in Italian) over four days.
- Days 1 and 2 are mainly on strength of solid bodies, days 3 and 4 are on motion.
- We'll look at day 3, which begins with a work by Galileo called *De Motu Locali* (On Local Motion).

Galileo's claim of novelty

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the free motion [naturalem motum] of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced. (153)

Uniform motion

Definition

By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal. (154)

Galileo next gives four "axioms." One is:

Axiom 2

In the case of one and the same uniform motion, the time required to traverse a greater distance is longer than the time required for a less distance. (154)

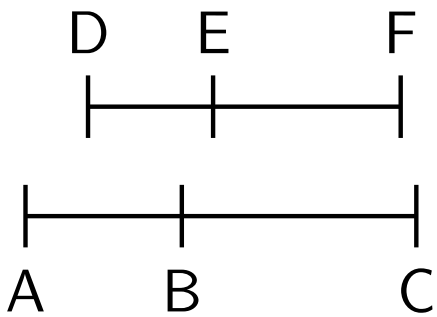
Questions

1. Galileo says he has discovered some properties of motion "which have not hitherto been either observed or demonstrated." Explain the difference between observing and demonstrating properties of motion.
2. How does Galileo define uniform motion?
3. State one of Galileo's "axioms". What would such "axioms" be called in Euclid's terminology, and in Aristotle's terminology? Explain.

Theorem 1

If a moving particle, carried uniformly at a constant speed, traverses two distances the time-intervals required are to each other in the ratio of these distances. (155)

Let a particle move uniformly with constant speed through two distances AB, BC, and let the time required to traverse AB be represented by DE; the time required to traverse BC, by EF; then I say that the distance AB is to the distance BC as the time DE is to the time EF. (155)



In modern notation: AB:BC = DE:EF.

Euclid's definition of equal ratios (*Elements* Book 5 Definition 5)

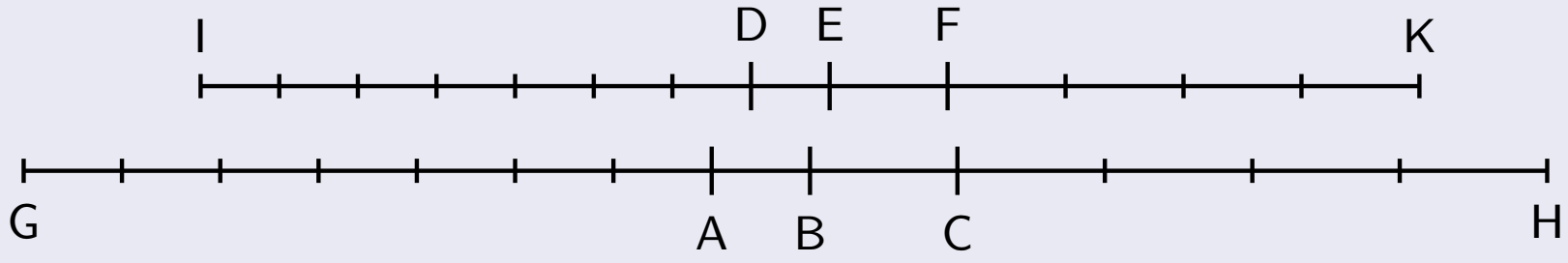
Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever are taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

This says that $a{:}b = c{:}d$ if, for all natural numbers m and n, either

- $ma > nb$ and $mc > nd$, or
- $ma = nb$ and $mc = nd$, or
- $ma < nb$ and $mc < nd$.

Proof of Theorem 1

Let GB and IE be m times AB and DE, respectively. Let BH and EK be n times BC and EF, respectively.



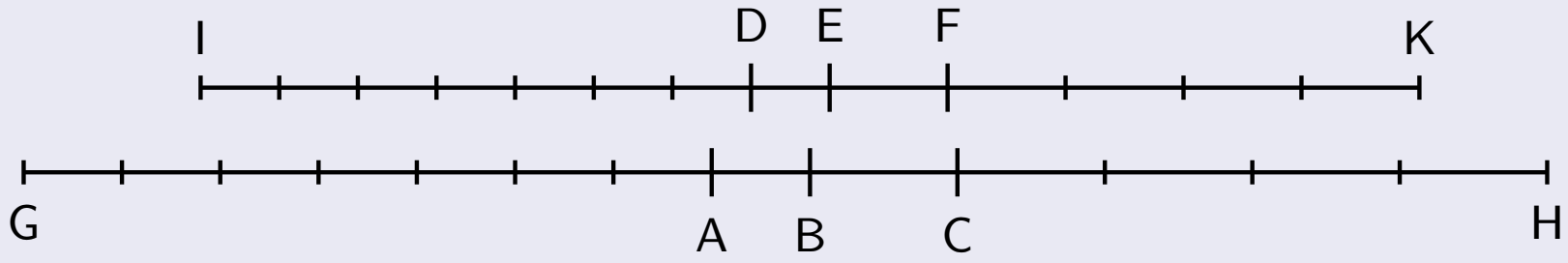
(a) The time required to traverse GB is IE

GB contains m intervals equal to AB. By the definition of uniform motion, the time to traverse each of those intervals is the same as the time to traverse AB, namely DE. Therefore, the time to traverse GB is mDE, which equals IE.

(b) The time required to traverse BH is EK

Proof similar to (a).

Proof of Theorem 1 (continued)



(c) If GB $>$ BH then IE $>$ EK

Suppose GB $>$ BH. By Axiom 2, the time to traverse GB is greater than to traverse BH. So, by (a) and (b), IE $>$ EK.

(d) If GB $=$ BH then IE $=$ EK

Suppose GB $=$ BH. By the definition of uniform motion, the time to traverse GB and BH is the same. So, by (a) and (b), IE $=$ EK.

(e) If GB $<$ BH then IE $<$ EK

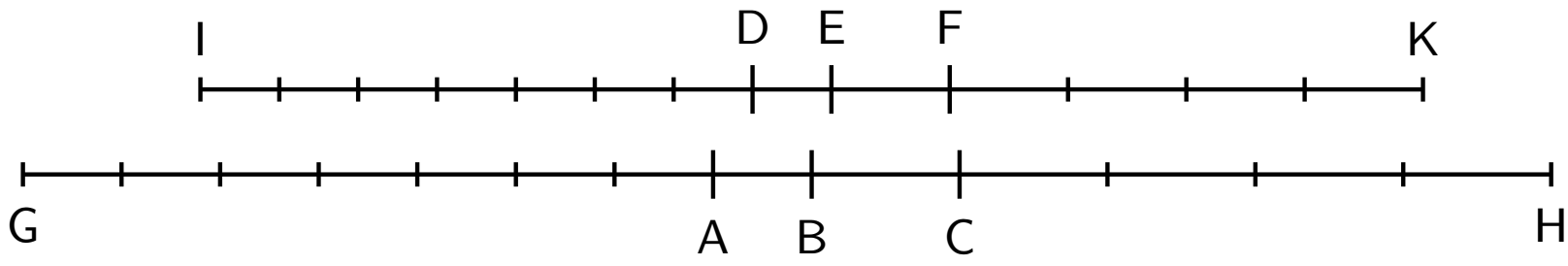
Proof similar to (c).

(f) AB:BC $=$ DE:EF

By (c), (d), and (e), if $m\text{AB} \gtreqless n\text{BC}$ then $m\text{DE} \gtreqless n\text{EF}$.
So, by Euclid's definition of equal ratios, AB:BC $=$ DE:EF.

Question

4. A particle moves uniformly through distances AB, BC, and the times to traverse these distances are DE and EF, respectively. Let GB and IE be m times AB and DE, respectively; let BH and EK be n times BC and EF, respectively.



Explain why the following statements are true; say where Galileo's definition and axioms for uniform motion are used.

(a) The time required to traverse GB is IE
(b) The time required to traverse BH is EK
(c) If GB $>$ BH then IE $>$ EK
(d) If GB $=$ BH then IE $=$ EK
(e) If GB $<$ BH then IE $<$ EK
(f) AB:BC $=$ DE:EF

Reference

Galileo Galilei.
Dialogues Concerning Two New Sciences.
Macmillan, 1914.
Translated by Henry Crew and Alfonso de Salvio.
Online in facsimile pdf (16MB) and html.
Numbers in parentheses are page numbers of this edition.