

Lecture 6

Euclid Propositions 2 and 3

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Scientific Thought I
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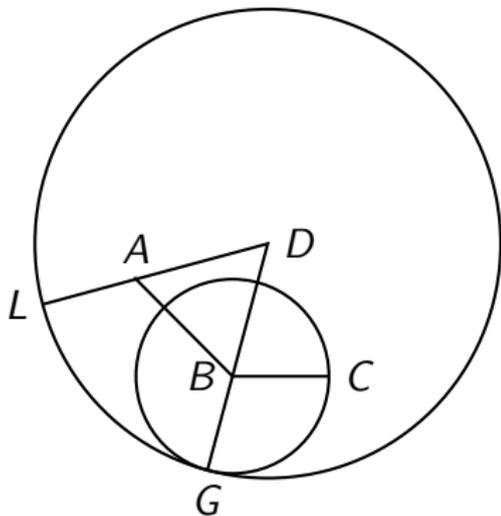
Proposition 2

To place at a given point [as an extremity] a straight line equal to a given straight line.

Construction:

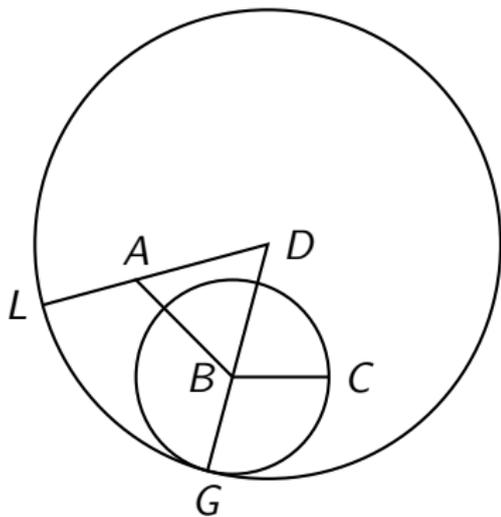
- 1 Let A be the given point and BC the given line.
- 2 Draw a straight line from A to B . [Post. 1]
- 3 Construct an equilateral triangle ABD on AB . [Prop. 1]
- 4 Draw a circle, center B , distance BC . [Post. 3]
- 5 Extend DB so it meets this circle at G . [Post. 2]
- 6 Draw a circle, center D , distance DG . [Post. 3]
- 7 Extend DA so it meets this circle at L . [Post. 2]

AL is the line we want.



Proof that the construction does what was required:

- 8 $DL = DG$. [Def. 15]
- 9 $DA = DB$. [Def. 20]
- 10 $AL = BG$. [C.N. 3]
- 11 $BG = BC$. [Def. 15]
- 12 $AL = BC$. [C.N. 1] Q.E.F.



- 1 Prove the following proposition from Euclid.

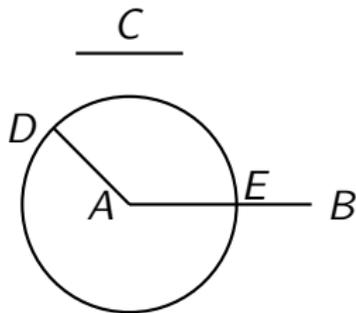
To place at a given point (as an extremity) a straight line equal to a given straight line.

You don't need to state the numbers of the definitions, postulates, etc., that you use but you should indicate where you are using one; e.g., you can write “[C.N.]” when you use a common notion.

Proposition 3

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.

- 1 Let AB be the longer line and C the shorter one.
- 2 Draw a line AD equal to C .
[Prop. 2]
- 3 Draw a circle, center A , distance AD . [Post. 3]
- 4 Let E be the point where this circle intersects AB .
- 5 $AE = AD$. [Def. 15]
- 6 $AD = C$. [construction]
- 7 $AE = C$. [C.N. 1] Q.E.F.



Comments

- What can be used in proofs: Definitions, postulates, common notions, *and previously proved propositions*.
- Proposition 1 is used in the proof of Proposition 2, which is used in the proof of Proposition 3. So Euclid had to put them in that order.
- Propositions 1–3 are all problems; the first theorem is Proposition 4.

- 2 Prove the following proposition from Euclid.

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.

You don't need to state the numbers of the definitions, postulates, etc., that you use but you should indicate where you are using one; e.g., you can write “[C.N.]” when you use a common notion.

References



Dana Denmore, editor.

Euclid's Elements.

Green Lion Press, 2002.

Heath's translation without his commentary.

There is a free [pdf file of Book I to Proposition 7](#).

My quotations are from this.



Thomas L. Heath.

Euclid's Elements.

Dover, 1956.

Classic edition, with extensive commentary, in 3 vols.



D. E. Joyce.

Euclid's Elements.

A web version with commentary and modifiable diagrams.