# Lecture 26
# Maher on the Law of Likelihood

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# Favoring

## Notation

$E$ = evidence
$H_i$ = incompatible hypotheses
$p(H)$ = ip of $H$ given only background evidence
$p(H|E)$ = ip of $H$ given $E$ in addition to background evidence

## Definition

$E$ **favors** $H_1$ over $H_2$ if

$$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)}.$$

In words: Evidence $E$ increases the ratio of the probability of $H_1$ to the probability of $H_2$ (where probability = ip).

## Examples

1. $p(H_1|E) = 0.2$, $p(H_2|E) = 0.1$, $p(H_1) = p(H_2) = 0.4$.

$$\frac{p(H_1|E)}{p(H_2|E)} = 2. \quad \frac{p(H_1)}{p(H_2)} = 1.$$

   $E$ favors $H_1$ over $H_2$.

2. $p(H_1|E) = 0.6$, $p(H_2|E) = 0.3$, $p(H_1) = 0.4$, $p(H_2) = 0.2$.

$$\frac{p(H_1|E)}{p(H_2|E)} = 2. \quad \frac{p(H_1)}{p(H_2)} = 2.$$

   $E$ doesn't favor $H_1$ over $H_2$ or $H_2$ over $H_1$.

## Note

"$E$ favors $H_1$ over $H_2$" doesn't imply $p(H_1|E) > 1/2$ or $p(H_1|E) > p(H_1)$.

# The law of likelihood

## The law

$E$ favors $H_1$ over $H_2$ if and only if $p(E|H_1) > p(E|H_2)$.

- Probabilities of the form $p(E|H_i)$ are called *likelihoods*.
- The law follows from the laws of probability and the definition of favoring (proof at the end).

## Examples

1. If $H_1$ implies $E$ and $H_2$ doesn't then $p(E|H_1) = 1$ and $p(E|H_2) < 1$, so $E$ favors $H_1$ over $H_2$.
2. If $H_1$ and $H_2$ both imply $E$ then $p(E|H_1) = 1$ and $p(E|H_2) = 1$, so $E$ doesn't favor either hypothesis over the other.

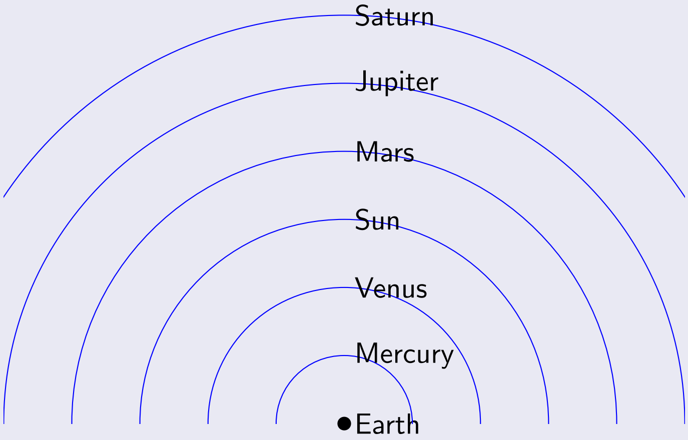
# Questions

1. State the definition of favoring and the law of likelihood.
2. For each of the following, say which (if either) of $H_1$ and $H_2$ is favored by $E$. Justify your answers using either the definition of favoring or the law of likelihood.
   (a) $p(H_1|E) = 0.7$, $p(H_2|E) = 0.2$, $p(H_1) = 0.5$, $p(H_2) = 0.1$.
   (b) $p(E|H_1) = 0.7$, $p(E|H_2) = 0.2$, $p(H_1) = 0.5$, $p(H_2) = 0.1$.
   (c) A ball is drawn from an urn. $H_1 =$ 10% of the balls in the urn are black, $H_2 =$ 20% of them are black, $E =$ the ball drawn is black.
   (d) A die is tossed. $H_1 =$ it came up 4 or 6, $H_2 =$ it came up 2, $E =$ it came up even.

# Application to Ptolemy and Copernicus

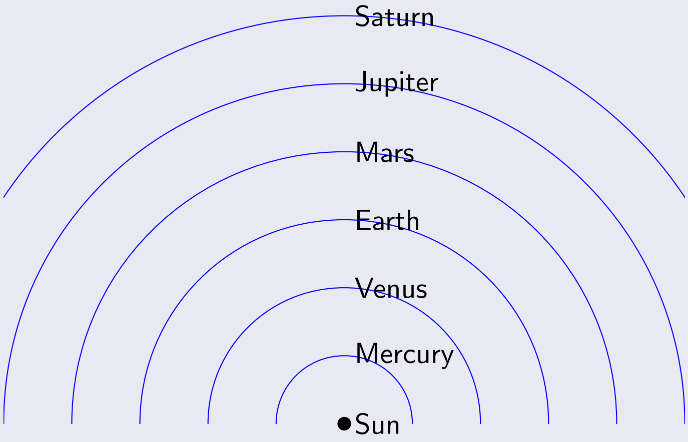
## $T$ (Ptolemy's claim)

The sun and planets orbit the earth, in the order shown.
Planets are on epicycles.



## $C$ (Copernicus's claim)

The planets, including the earth, orbit the sun in the order shown. No epicycles.

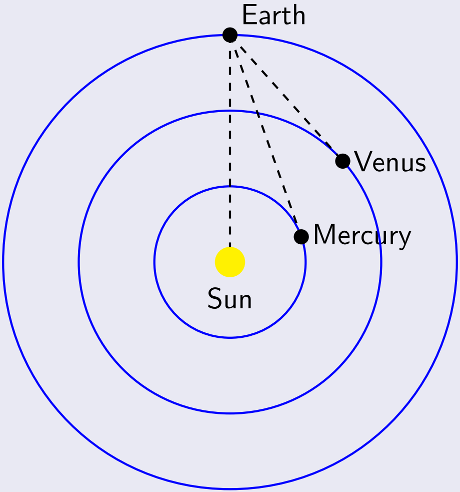


### $E_1$: Mercury and Venus always appear close to the Sun

- Mercury is never more than $28^\circ$ from the Sun; Venus is never more than $48^\circ$.
- Other planets can be on the opposite side of the sky to the Sun; Mercury and Venus never are.

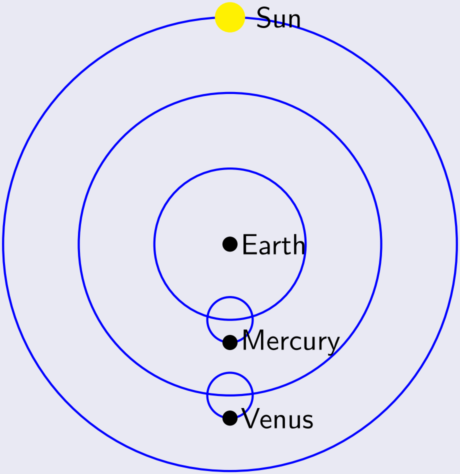
## $C$ implies $E_1$

Since Mercury and Venus orbit the sun with a smaller orbit than the earth's, they can never appear far from the sun.



## $T$ doesn't imply $E_1$

This is consistent with $T$:



## Application of law of likelihood

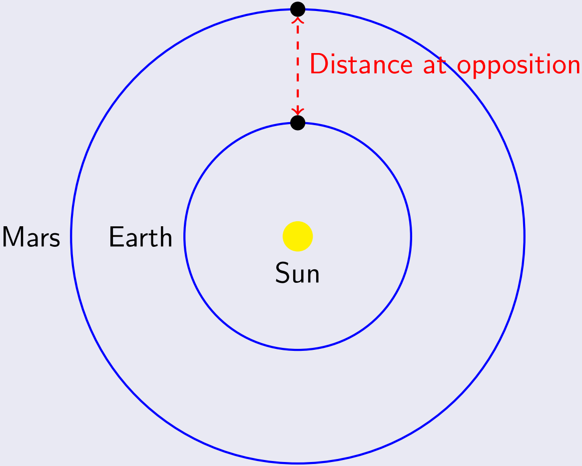
- $p(E_1|C) = 1$, since $C$ implies $E_1$.
- $p(E_1|T) < 1$, since $T$ doesn't imply $E_1$.
- So $p(E_1|C) > p(E_1|T)$.
- So, by the law of likelihood, $E_1$ favors $C$ over $T$.

## $E_2$: Superior planets are closest when in opposition to the sun

- The superior planets are Mars, Jupiter, and Saturn.
- A planet is in opposition to the sun when it is on the opposite side of the sky to the sun.
- We know the superior planets are closest when in opposition because that is when they are brightest.

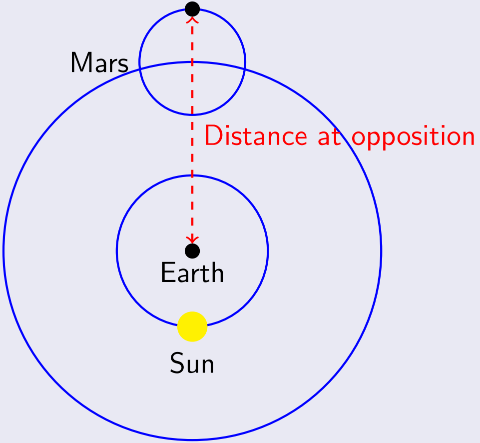
## $C$ implies $E_2$

When a superior planet is in opposition to the sun, the earth is between the planet and the sun, which minimizes the distance to the planet.



## $T$ doesn't imply $E_2$

This is consistent with $T$:



## Application of law of likelihood

- $p(E_2|C) = 1$, since $C$ implies $E_2$.
- $p(E_2|T) < 1$, since $T$ doesn't imply $E_2$.
- So $p(E_2|C) > p(E_2|T)$.
- So, by the law of likelihood, $E_2$ favors $C$ over $T$.

# Question

3. Let $T$ = Ptolemy's claim that the sun and planets orbit the earth on epicycles, $C$ = Copernicus's claim that the earth and other planets orbit the sun. Which of these is favored by the following pieces of evidence? Justify your answer using the law of likelihood; draw diagrams as appropriate.

   $E_1$: Mercury and Venus always appear close to the sun.

   $E_2$: The superior planets are closest to the earth when in opposition to the sun.

# Proof of the law of likelihood (not on the exam)

Let $H_1, \dots, H_n$ be an exhaustive set of incompatible hypotheses. By a law of probability called Bayes's theorem:

$$p(H_i|E) = \frac{p(E|H_i)p(H_i)}{p(E|H_1)p(H_1) + \cdots + p(E|H_n)p(H_n)}.$$

Dividing the expression for $i = 1$ by the one for $i = 2$ gives:

$$\frac{p(H_1|E)}{p(H_2|E)} = \frac{p(E|H_1)}{p(E|H_2)} \frac{p(H_1)}{p(H_2)}.$$

It follows that

$$\frac{p(H_1|E)}{p(H_2|E)} > \frac{p(H_1)}{p(H_2)} \text{ if and only if } \frac{p(E|H_1)}{p(E|H_2)} > 1.$$

So, by the definition of favoring, $E$ favors $H_1$ over $H_2$ if and only if $p(E|H_1) > p(E|H_2)$.