Lecture 2
Euclid

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- Euclid was alive and working around 300 BC.
- He established a school in Alexandria, a city in Egypt that had been founded by Alexander the Great.
- He put together geometrical work of many predecessors into a comprehensive and unified treatise, the *Elements*.
- This quickly became the main textbook of geometry and continued to be that for more than 2000 years.
- Geometry as presented by Euclid has been a model of science from the ancient Greeks to recent times.
- We'll discuss the first few pages of Book I, which deals with trigonometry.

Euclid begins with definitions. They explain the meaning of
geometrical terms used in his book.

Some of Euclid's definitions

1. *A point is that which has no part.*
2. *A line is breadthless length.*
4. *A straight line is a line which lies evenly with the points on itself.*
10. *When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.*

Some more definitions

15. A *circle* is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;

16. And the point is called the *centre* of the circle.

19. *Rectilineal figures* are those which are contained by straight lines, *trilateral* figures being those contained by three, quadrilateral those contained by four, and *multilateral* those contained by more than four straight lines.

20. Of trilateral figures, an *equilateral triangle* is that which has its three sides equal, an *isosceles triangle* that which has two of its sides alone equal, and a *scalene triangle* that which has its three sides unequal.

Postulates

Postulates are things assumed without proof concerning geometry.

Euclid's postulates

1. *To draw a straight line from any point to any point.*
2. *To produce a finite straight line continuously in a straight line.*
3. *To describe a circle with any centre and distance.*
4. *That all right angles are equal to one another.*
5. *That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.*

- In Postulate 3, the "distance" is from the center of the circle to a point on the circumference, i.e., the radius of the circle.
- Postulates 1–3 say that certain constructions can be done.
- Postulates 4 and 5 say certain things have certain properties.

Common notions

Common notions are like postulates in being assumed without proof. The difference is that postulates are concerned with geometrical matters whereas common notions are general principles that aren't limited to geometry—they are "common" in the sense that they apply to other subjects as well.

Euclid's common notions

1. *Things which are equal to the same thing are also equal to one another.*
2. *If equals be added to equals, the wholes are equal.*
3. *If equals be subtracted from equals, the remainders are equal.*
4. *Things which coincide with one another are equal to one another.*
5. *The whole is greater than the part.*

Propositions are things proved based on the definitions, postulates, and common notions. There are two kinds:

1. Some propositions state something to be done. Examples:
   - Proposition 1: *On a given finite straight line to construct an equilateral triangle.*
   - Proposition 2: *To place at a given point [as an extremity] a straight line equal to a given straight line.*

2. Other propositions say something is the case. Example:
   - Proposition 5: *In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.*

Terminology for the two kinds of proposition

- Propositions of the first type are called **problems**; those of the second type are called **theorems**.
- Euclid didn't use that terminology; he marked the difference by how he ends the proof. Proofs of problems end:
    - "Being what it was required to do"
    - In Latin: Quod erat faciendum (Q.E.F.)

    Proofs of theorems end:
    - "Being what it was required to demonstrate"
   - In Latin: Quod erat demonstrandum (Q.E.D.)

1. In Euclid, how does a postulate differ from (a) a definition, (b) a common notion, and (c) a proposition?
2. What are the two kinds of proposition called and what is the difference between them?
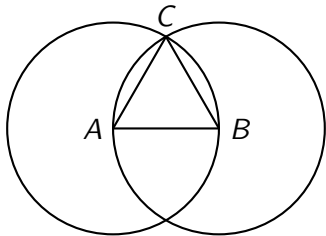
Proposition 1

On a given finite straight line to construct an equilateral triangle.

First part of the proof: describe the construction.

1. Let *AB* be the given line.
2. Draw a circle, center *A*, distance *AB*. [Post. 3]
3. Draw a circle, center *B*, distance *AB*. [Post. 3]
4. Let *C* be a point where the circles intersect.
5. Draw straight lines *AC* and *BC*. [Post. 1]
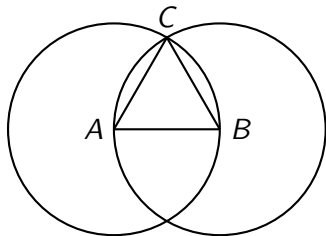
ABC is the required triangle.

Second part: Prove that the construction does what was required.

- ⑥ $AB = AC$ [Def. 15]
- ⑦ $AB = BC$ [Def. 15]
- ⑧ $AC = BC$ [C.N. 1]
- ⑨ Hence ABC is an equilateral triangle [Def. 20]
  and it is on the given line AB. Q.E.F.

3 Prove the following proposition from Euclid:

On a given finite straight line to construct an equilateral triangle.

You don't need to state the numbers of the definitions, postulates, etc., that you use but you should indicate where you are using one; e.g., you can write "[C.N.]" when you use a common notion.

📕 Dana Densmore, editor.
Euclid's Elements.
Green Lion Press, 2002.
Heath's translation without his commentary.
There is a free pdf file of Book I to Proposition 7.
My quotations are from this.

📕 Thomas L. Heath.
Euclid's Elements.
Dover, 1956.
Classic edition, with extensive commentary, in 3 vols.

📕 D. E. Joyce.
Euclid's *Elements*.
A web version with commentary and modifiable diagrams.